where

R = the gas constant in the expression.

$$pv = RT \tag{21}$$

k = the ratio of the specific heat at constant pressure to the specific heat at constant volume.

This ratio, k, is used extensively in fluid dynamics; values of k for various gases are given in Table 2.

TABLE 2. RATIO OF SPECIFIC HEAT AT CONSTANT PRESSURE TO SPECIFIC HEAT AT CONSTANT VOLUME FOR COMPRESSIBLE FLUIDS

COMPRESSIBLE FLUID	RATIO $k = c_p/c_v$
Helium and other monatomic gases	1.66
Air and other diatomic gases	1.40
Ammonia and hydrogen sulfide	1.34
Carbon dioxide, methane, natural gas, superheated steam, moist steam down to a quality of 97 per cent	1.28 to 1.32
Sulfur dioxide, ethylene, acetylene	1.24 to 1.26

Substituting Equations 20 and 21 in Equation 19, the energy equation becomes

$$\frac{V_2^2 - V_1^2}{2g} = \frac{k}{k-1}(p_1v_1 - p_2v_2) \tag{22}$$

While this is a convenient form of equation, it does not include all the necessary specifications. If the steady flow process is frictionless and reversible,

$$\frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^k = \left(\frac{\rho_2}{\rho_1}\right)^k \tag{23}$$

By introducing this relation in Equation 22 it is possible to reduce that equation to:

$$\frac{V_2^2 - V_1^2}{2g} = \frac{k}{k-1}\frac{p_1}{\rho_1}\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}\right] \tag{24}$$

This form of the equation is applicable not only to flow in pipes, but also to flow through orifices and nozzles.

A significant factor in the flow of compressible fluids is the velocity of sound, V_{so}, which for present purposes will be considered as that velocity under the conditions at the entrance to the steady flow system. The velocity of sound is expressed as

$$V_{so} = \sqrt{\frac{gkp_1}{\rho_1}} \tag{25}$$

This formula may be developed rationally and agrees perfectly with experimental results. Substituting Equation 25 in Equation 24:

$$\frac{V_2^2 - V_1^2}{2} = \frac{V_{so}^2}{k-1}\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}\right] \tag{26}$$

or, by rearrangement,

$$\frac{p_2}{p_1} = \left[1 - \frac{k-1}{V_{so}^2}\left(\frac{V_2^2 - V_1^2}{2}\right)\right]^{\frac{k}{k-1}} \tag{27}$$

which permits the calculation of the ratio of pressures at entrance and exit of the steady flow device,—pipe, orifice, or nozzle.

FLOW THROUGH NOZZLE OR ORIFICE

Another useful expression, covering the energy change in an orifice or nozzle, may be derived from Equation 2. As with the flow through pipes, no outside work is done. Then, assuming that there is no difference in elevation, and since practically no heat is evolved or absorbed, *i.e.*, the process is adiabatic, E, z, and q of Equation 2 may be eliminated, and, by rearranging, the equation becomes

$$\frac{V_2^2}{2g} - \frac{V_1^2}{2g} = J(h_1 - h_2) \text{ foot pounds per second} \tag{28}$$

In any flow device,

$$\frac{V_1A_1}{v_1} = \frac{V_2A_2}{v_2} \; or, \; V_1 = V_2\frac{A_2v_1}{A_1v_2} \tag{29}$$

in which A_1 or A_2 is the cross-sectional area of the flow at a particular point, expressed in square feet. With this substituted in Equation 28, and solving for V_2:

$$V_2 = \sqrt{\frac{2gJ(h_1 - h_2)}{1 - (A_2/A_1)^2 (v_1/v_2)^2}} \tag{30}$$

Using this expression, it is possible to determine the velocity at any point in the flow through an orifice or nozzle. If the area at the point of entry is very large with respect to that at point 2, the denominator on the right side of Equation 30 will approach unity, and the equation will reduce to

$$V_2 = \sqrt{2gJ(h_1 - h_2)} \tag{31}$$

For this reason, the expression $\sqrt{\dfrac{1}{1 - (A_2/A_1)^2 (v_1^2/v_2^2)}}$ is called the correction factor for the velocity of approach.

The velocity of approach factor may be further simplified if the difference in volume between points 1 and 2 is negligible. Under this condition, the velocity of approach factor becomes $\sqrt{\dfrac{1}{1 - (A_2/A_1)^2}}$. If

$$\frac{A_2}{A_1} = \frac{D_2^2}{D_1^2} = \beta^2 \tag{32}$$

the velocity of approach factor is $\sqrt{\dfrac{1}{1 - \beta^4}}$, in which form it is generally used in flow formulas. The quantity β is the ratio of the throat or orifice diameter to the pipe diameter.