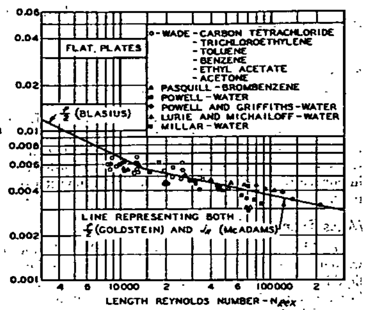

Fig. 7 .... Mass Transfer from a Flat Plate

used for this forced convection mass transfer process. At least, at low mass transfer rates, where v_i is small, the factor should be adequate. Experiments have proved the assumption to be justified as will be shown below. Therefore, the total mass transfer rate from the interface may be written as

$$\dot{m}_v/A = \frac{-D_v P_t}{R_v T p_{am}}\left(\frac{dp_v}{dy}\right)_i \tag{20}$$

The concentration gradient at the interface $(dp_v/dy)_i$ must be evaluated experimentally. Rather than work with this gradient, it is the common practice to define a mass transfer coefficient, as in Equation 2. It follows that

$$\dot{m}_v/A = (h_D/R_v T)(p_{vi} - p_{v\infty}) = \frac{-D_v P_t}{R_v T p_{am}}\left(\frac{dp_v}{dy}\right)_i$$

By some mathematical rearrangement this becomes

$$N_{Sh} = \frac{h_D L p_{am}}{D_v P_t} = \left[\frac{d\left(\frac{p_v - p_{vi}}{p_{v\infty} - p_{vi}}\right)}{d(y/L)}\right]_i \tag{21}$$

where N_{Sh} is the dimensionless Sherwood number and L is some characteristic dimension of the mass transfer surface such as the length of a plate or diameter of a cylinder. Equation 21 gives a simple, but valuable, physical interpretation of the dimensionless mass transfer coefficient, i.e., the Sherwood number. It is seen to be the dimensionless concentration gradient at the interface or mass transfer boundary. The gradient is illustrated in Fig. 6.

Mass transfer coefficients have been established experimentally for a number of flow geometries, and are presented in the literature on this subject.[3-6] However, data on mass transfer are relatively sparse as compared to the wealth of data on heat transfer coefficients. Recognizing that there is an analogy between these transfer processes, it should be possible to use these heat transfer data in predicting mass transfer coefficients. Such similarity relations do exist, and their reliability has been well established at low mass transfer rates.

Similarity Relations for Convective Mass Transfer

Heat transfer from the wetted surface of Fig. 6 to the air stream can be expressed in an exactly analogous form to that of the mass transfer. Thus, the heat flux per unit area is

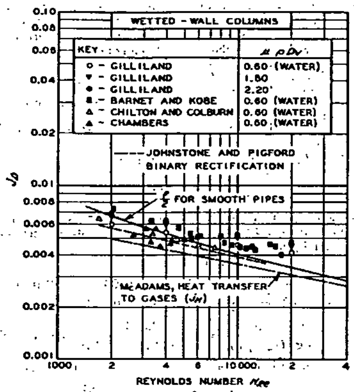

Fig. 8 .... Vaporization and Absorption in a Wetted Wall Column

$$q/A = -k\left(\frac{dt}{dy}\right)_i = h(t_i - t_\infty)$$

or, by mathematical rearrangement

$$N_{Nu} = \frac{hL}{k} = \left[\frac{d\left(\frac{t - t_i}{t_\infty - t_i}\right)}{d(y/L)}\right]_i \tag{22}$$

The familiar dimensionless Nusselt number (N_{Nu}) of heat transfer is now recognized as the dimensionless temperature gradient at the heat transfer boundary (interface). Furthermore, the Sherwood and Nusselt numbers are recognized as analogous expressions for dimensionless mass transfer and heat transfer coefficients, respectively.

Again, in the case of momentum transfer, it is the usual practice to define a friction factor f such that the momentum flux at the interface $(g\tau_i)$ per unit area is

$$g\tau_i = -\mu\left(\frac{dV_x}{dy}\right)_i = f\frac{\rho V_\infty^2}{2}$$

or

$$\frac{f}{2}\left(\frac{\rho V_\infty L}{\mu}\right) = \left[\frac{d(V_x/V_\infty)}{d(y/L)}\right]_i \tag{23}$$

where

V_∞ = mean or free stream velocity of gas, feet per hour.

Equation 23 states that one-half the friction factor times the Reynolds number $(N_{Re} = \rho V_\infty L/\mu)$ equals the dimensionless velocity gradient at the stationary surface or interface.

Equations 21, 22, and 23 show the similarities between mass, heat, and momentum transport in a turbulent boundary layer (where both eddy and molecular diffusion are operative), in the same way that Equations 8, 9, and 10 demonstrated the similarities for a laminar boundary layer (where only molecular diffusion is present). Again assume a gaseous system in which, by chance, $D_v = \alpha = \nu$ (that is, the molecular diffusion constants for mass, energy, and momentum are identical). From a purely theoretical consideration of the

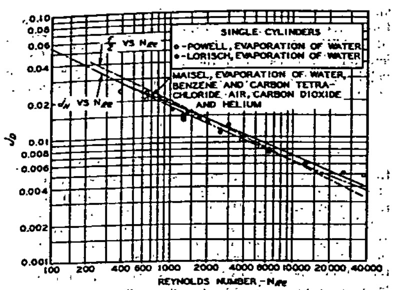

Fig. 9 .... Mass Transfer from Single Cylinders in Crossflow

physical process of eddy diffusion, the eddy diffusivities for these three rate processes must be identical, since all rely on the same particle mixing action for the transfer operation. Experiments verify this conclusion. Therefore, under the previous assumption, the dimensionless partial pressure, temperature, and velocity profiles must be identical. This permits equating the left-hand side of Equations 21, 22, and 23 to obtain $N_{Sh} = N_{Nu} = (f/2)(N_{Re})$ or

$$\frac{h_D L p_{am}}{D_v P_t} = \frac{hL}{k} = \frac{f}{2}\frac{V_\infty L}{\nu} \tag{24}$$

As pointed out in the discussion of molecular diffusion, the postulate made here is equivalent to the statement that the Prandtl number $(N_{Pr} = c_p\mu/k)$ and Schmidt number $(N_{Sc} = \mu/\rho D_v)$ are unity. Using this statement Equation 24 can be rearranged to give

$$\frac{N_{Sh}}{(N_{Re})(N_{Sc})} = \frac{N_{Nu}}{(N_{Re})(N_{Pr})} = \frac{f}{2} \tag{24a}$$

or

$$\frac{h_D p_{am}}{V_\infty P_t} = \frac{h}{\rho c_p V_\infty} = \frac{f}{2} \tag{24b}$$

The two right-hand terms of Equation 24 are known as the *Reynolds analogy*. The left-hand term is an extension of Reynolds analogy to include mass transfer.

Investigators of convective heat transfer have long recognized that Reynolds analogy gave a reliable correlation between friction factors and heat transfer coefficients for common gases (where $N_{Pr} \cong 1$), and where the temperature potential is moderate. Since the time of Reynolds, not less than five modified analogies have been suggested to account for the effect of Prandtl number. One of the simplest and most often used among these is that suggested by Chilton and Colburn[7] who showed that the correlation of heat transfer data with friction data could be greatly improved by substituting $(N_{Pr})^{1/3}$ for N_{Pr} in Equation 24a which leads to

$$j_H = \left(\frac{h}{\rho c_p V_\infty}\right)\left(\frac{c_p\mu}{k}\right)^{2/3} = \frac{f}{2} \tag{25}$$

By analogy, Chilton and Colburn suggested that it should be possible to represent mass transfer data with good accuracy by a similar change in the Schmidt number parameter, hence

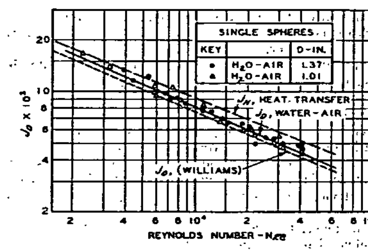

Fig. 10 .... Mass Transfer from Single Spheres

$$j_D = \left(\frac{h_D p_{am}}{V_\infty P_t}\right)\left(\frac{\mu}{\rho D_v}\right)^{2/3} = \frac{f}{2} \tag{26}$$

Equations 25 and 26 are known as the *j-factor analogy* and are widely used today for plotting and predicting heat and mass transfer data.

The power of the Chilton-Colburn *j*-factor analogy is represented in Figs. 7, 8, 9, and 10. Fig. 7 is a plot of experimental values of j_D by a number of investigators of mass transfer from a flat plate with flow parallel to the plate surface. The solid line, which represents the data to near perfection, is actually $f/2$ from Blasius solution of laminar flow on a flat plate (left hand portion of the solid line) and Goldstein's solution for a turbulent boundary layer (right hand portion). The right hand portion of the solid line also represents McAdams[8] correlation of turbulent flow heat transfer coefficients for a flat plate.

A wetted-wall column is a vertical tube in which a thin liquid film adheres to the tube surface and exchanges mass by evaporation or absorption with a gas flowing through the tube. It is a device of practical interest and provides one of the best tests of transport theories. Fig. 8 illustrates typical data on vaporization in wetted-wall columns, plotted as j_D vs N_{Re}. The spread of the points with variation in $\mu/\rho D_v$ is due to the fact that Gilliland found an exponent of 0.56, not 2/3, as representing the effect of the Schmidt number.[3] This is shown by the fact that Gilliland's equation may be written

$$j_D = 0.023 N_{Re}^{-0.17}(\mu/\rho D_v)^{0.11} \tag{27}$$

Similarly McAdams[8] equation for heat transfer in pipes may be expressed as

$$j_H = 0.023(N_{Re})^{-0.20}(c_p\mu/k)^{0.67} \tag{28}$$

This is represented by the dashed curve shown on Fig. 8, which falls somewhat below the mass transfer data. The curve $f/2$ representing friction in smooth tubes is the upper solid curve.

Data for evaporation of liquids from single cylinders into gas streams flowing transversely to the cylinder axis are shown in Fig. 9. Although the dash-dot line on Fig. 9 represents the data well, it is actually taken from McAdams[8] as representative of a large collection of data on heat transfer to single cylinders placed transverse to air streams. In order to compare these data with friction, it is necessary to distinguish between total drag and skin friction. The analogies are based on skin friction, hence the normal pressure drag must be subtracted from the measured total drag. At $N_{Re} = 1000$ the skin friction is 12.6 percent of the total drag and at N_{Re}