

BP-BPb / Misc. Papers

TEH 0412751

N 31038

# MIXED EFFECTS MODELS FOR CATEGORICAL DATA

J. Richard Landis

Department of Biostatistics, School of Public Health,  
University of Michigan, Ann Arbor, MI 48109, U.S.A.

James M. Lepkowski

Sampling Section of the Survey Research Center, Institute for Social  
Research and Department of Biostatistics, School of Public Health,  
University of Michigan, Ann Arbor, MI 48109, U.S.A.

Paula J. Beitler

Ortho Pharmaceutical Corp., Route 202 South,  
Raritan, NJ 08869, U.S.A.

## *Abstract*

Several mixed effects models for categorical data from unbalanced designs are proposed. The analogy between these models and a clustered sampling model is discussed. Variance components are estimated by an extension of the fitting constants method for the mixed ANOVA models and through the estimation of design effects for the cluster sampling model.

These methods are illustrated using health research data from two different contexts. First, the results from a multi-center randomized clinical trial are analyzed under the assumptions that the treatment effects are fixed and the clinics are presumed to be random. Secondly, the application of these methods to data obtained by cluster sampling is illustrated with data from the second National Health and Nutrition Examination Survey (NHANES II). In particular, the relationship between elevated blood lead levels ( $\geq 15$   $\mu\text{g/dl}$ ) and elevated diastolic blood pressure ( $\geq 90$  mm Hg) is modeled across the 64 sampling sites for 3,181 males ages 12-74.

N 31038.01

TEH 0412752

## Notation for Binomial Data (Two-way Model)

| Notation                  | Role                                                                           |
|---------------------------|--------------------------------------------------------------------------------|
| $a = 1, 2, \dots, A$      | indexes the 1st stage clusters(clinics)                                        |
| $i = 1, 2, \dots, I$      | indexes the fixed effects(treatments)                                          |
| $k = 1, 2, \dots, n_{ai}$ | indexes the subjects within the $a$ th clinic assigned to the $i$ th treatment |

## Indicator Variables

$$y_{aik} = \begin{cases} 1, & \text{if the } k\text{th subject in the } a\text{th clinic} \\ & \text{assigned to the } i\text{th treatment responds} \\ & \text{favorably} \\ 0, & \text{otherwise} \end{cases}$$

## Sample Sizes and Proportions

| Notation                            | Description                                                                              |
|-------------------------------------|------------------------------------------------------------------------------------------|
| $n_i = \sum_a n_{ai}$               | total number of subjects assigned to the $i$ th treatment                                |
| $p_{ai} = \sum_k y_{aik} / n_{ai}$  | sample proportions responding favorably to the $i$ th treatment within the $a$ th clinic |
| $p_i = \sum_a \sum_k y_{aik} / n_i$ | overall sample proportion responding favorably to the $i$ th treatment                   |

May 11, 1985

ENAR Mtgs., Raleigh, NC, 3/25/85

TEH 0412753

## Means &amp; Variances: Indicator Variables

| Notation                          | Description                                                                         |
|-----------------------------------|-------------------------------------------------------------------------------------|
| $E\{y_{aik}\} = \pi_i$            | overall probability of favorable response on $i$ th treatment                       |
| $Var\{y_{aik}\} = \pi_i(1-\pi_i)$ | "element" variance for the indicator variables associated with the $i$ th treatment |

## Covariances &amp; Correlations: Indicator Variables

| Notation                                              | Description                                                                  |
|-------------------------------------------------------|------------------------------------------------------------------------------|
| Same Clinic, Same Treatment, Different Subjects       |                                                                              |
| $\sigma_{ii} = Cov\{y_{aik}, y_{aik'}\}$              | "element" covariance for the $i$ th treatment                                |
| $\rho_{ii} = \sigma_{ii}/\sigma_i^2$                  | (within clinic) intraclass correlation for the $i$ th treatment              |
| Same Clinic, Different Treatments, Different Subjects |                                                                              |
| $\sigma_{ii'} = Cov\{y_{aik}, y_{ai'k'}\}$            | "element" covariance between treatments $i$ and $i'$                         |
| $\rho_{ii'} = \sigma_{ii'}/\sigma_i\sigma_{i'}$       | (within clinic) intraclass cross-correlation between treatments $i$ and $i'$ |

Variances & Covariances: Sample Proportions Within Clinics  
(conditional on fixed sample sizes  $n_{ai}$ )

$$\text{Var}\{p_{ai}\} = \frac{\pi_i(1-\pi_i)}{n_{ai}} \left[ 1 + (n_{ai} - 1)\rho_{ii} \right]$$

$$\text{Cov}\{p_{ai}, p_{ai'}\} = \rho_{ii'} \sigma_i \sigma_{i'} = \sigma_{ii'}$$

Variances & Covariances: Overall Sample Proportions  
(conditional on fixed sample sizes  $n_i$ )

$$\text{Var}\{p_i\} = \frac{\pi_i(1-\pi_i)}{n_i} \left[ 1 + \left( \frac{\sum_a n_{ai}^2}{n_i} - 1 \right) \rho_{ii} \right]$$

$$\text{Cov}\{p_i, p_{i'}\} = \frac{\sum_a n_{ai} n_{ai'}}{n_i n_{i'}} \rho_{ii'} \sigma_i \sigma_{i'}$$

## Matrix Notation for 1 = 2 Treatments

| Notation                                                              | Description                                                                    |
|-----------------------------------------------------------------------|--------------------------------------------------------------------------------|
| $\pi_a = \begin{bmatrix} \pi_{a1} \\ \pi_{a2} \end{bmatrix}$          | vector of probabilities of favorable response within the ath clusters(clinics) |
| $\pi = \begin{bmatrix} \pi_1 \\ \pi_2 \end{bmatrix}$                  | vector of overall probabilities of favorable response for each treatment       |
| $p_a = \begin{bmatrix} p_{a1} \\ p_{a2} \end{bmatrix}$                | vector of sample proportions of favorable response within ath cluster(clinic)  |
| Sample Proportion Vector                                              |                                                                                |
| $p' = (p_{11}, p_{12}, \dots, p_{a1}, p_{a2}, \dots, p_{A1}, p_{A2})$ |                                                                                |

## Covariance Matrix for Sample Proportions in ath Cluster(Clinic)

$$V(p_a) = \begin{bmatrix} \frac{\sigma_1^2}{n_{a1}} \left[ 1 + (n_{a1} - 1)\rho_{11} \right] & \rho_{12} \sigma_1 \sigma_2 \\ \rho_{12} \sigma_1 \sigma_2 & \frac{\sigma_2^2}{n_{a2}} \left[ 1 + (n_{a2} - 1)\rho_{22} \right] \end{bmatrix}$$

TEH 0412756

ENAR Mtgs., Raleigh, NC, 3/25/85

May 11, 1985

## Mixed Effects ANOVA Model With Interaction

$$y_{aik} = \mu + c_a + \beta_i + g_{ai} + e_{aik}$$

$$\text{cov}(p_{ai}, p_{a'i'}) = \begin{cases} \sigma_{e1}^2/n_{ai} + \sigma_a^2 + \sigma_g^2, & a=a', i=i' \\ \sigma_a^2, & a=a', i \neq i' \\ 0 & \text{otherwise.} \end{cases}$$

$$V(p_a) = \begin{bmatrix} \frac{\sigma_{e1}^2}{n_{a1}} + \sigma_a^2 + \sigma_\gamma^2 & \sigma_a^2 \\ \sigma_a^2 & \frac{\sigma_{e2}^2}{n_{a2}} + \sigma_a^2 + \sigma_\gamma^2 \end{bmatrix}$$

May 11, 1985

ENAR Mtgs., Raleigh, NC, 3/25/85

TEH 0412757

DUP050453813

## Mixed Effects ANOVA Model Without Interaction

$$y_{aik} = \mu + c_a + \beta_i + e_{aik}$$

$$\text{cov}(p_{ai}, p_{a'i'}) = \begin{cases} \sigma_{ej}^2/n_{ai} + \sigma_a^2, & a=a', i=i' \\ \sigma_a^2, & a=a', i \neq i' \\ 0, & \text{otherwise.} \end{cases}$$

$$V(p_a) = \begin{bmatrix} \frac{\sigma_{e1}^2}{n_{a1}} + \sigma_a^2 & \sigma_c^2 \\ \sigma_a^2 & \frac{\sigma_{e2}^2}{n_{a2}} + \sigma_a^2 \end{bmatrix}$$

Estimated Covariance Matrix Based on Fixed Effects Multinomial Model  
(Unrestricted Covariance Matrix for Sample Proportions Within Clinic)

$$v(p_a) = \begin{bmatrix} \frac{p_{a1}(1-p_{a1})}{n_{a1}} & 0 \\ 0 & \frac{p_{a2}(1-p_{a2})}{n_{a2}} \end{bmatrix}$$

TEH 0412758

ENAR Mtgs., Raleigh, NC, 3/25/85

May 11, 1985

Estimated Covariance Matrix Based on Fixed Effects Multinomial Model  
( $H_0$  Covariance Matrix for Sample Proportions Within Cli)

$$v(p_a) = \bar{p}_a(1-\bar{p}_a) \begin{bmatrix} \frac{1}{n_{a1}} & 0 \\ 0 & \frac{1}{n_{a2}} \end{bmatrix}$$

Fixed Effects Models (A = 8 Clinics) (Reduced Model (Main Effects))

$$E\{p\} = X_R \beta_R = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 \\ 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \end{bmatrix} \begin{bmatrix} \pi \\ \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ \alpha_6 \\ \alpha_7 \end{bmatrix}$$

May 11, 1985

ENAR Mtgs., Raleigh, NC, 3/25/85 TEH 0412759

## Mixed Effects Model (A = 8 Clinics)

$$E\{p\} = X_M \beta_M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \pi_1 \\ \pi_2 \end{bmatrix}$$

## Cluster Sampling Model

| Quantity                                                 | Description                                                                                                                        |
|----------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------|
| Design Effect                                            | ratio of the variance of the proportion favorable under clustered design to the variance under simple random sampling assumptions. |
| $Deff_i = \frac{\frac{Var\{p_i\}}{\pi_i(1-\pi_i)}}{n_i}$ | design effect for response to $i$ th treatment due to regarding clinics as random sample.                                          |

TEH 0412760

ENAR Mtgs., Raleigh, NC, 3/25/85

May 11, 1985

Variances & Covariances: Overall Sample Proportions  
(conditional on fixed sample sizes  $n_i$ )

$$\text{Var}\{p_i\} = \frac{\pi_i(1-\pi_i)}{n_i} \left[ 1 + \left( \frac{\sum a_{ai}^2}{n_i} - 1 \right) \rho_{ii} \right]$$

$$\text{Cov}\{p_i, p_{i'}\} = \frac{\sum a_{ai} a_{ai'}}{n_i n_{i'}} \rho_{ii'} \sigma_i \sigma_{i'}$$

Cluster Sampling Model (Cont'd)  
Synthetic Estimators

$$\text{Deff}_i = 1 + \left( \frac{\sum a_{ai}^2}{n_i} - 1 \right) \rho_{ii}$$

$$\rho_{ii} = \frac{\text{Deff}_i - 1}{\frac{\sum a_{ai}^2}{n_i} - 1}$$

$$\rho_{12} = \frac{\text{Cov}\{p_1, p_2\}}{\frac{\sum a_{a1} a_{a2}}{n_1 n_2} \sigma_1 \sigma_2}$$

## Estimated Covariance Matrix Based on Complex Sample Estimators

$$v(p_a) = \begin{bmatrix} \frac{p_1(1-p_1)}{n_{a1}} \left[ 1 + (n_{a1}-1)\hat{\rho}_{11} \right] & \left[ p_1(1-p_1)p_2(1-p_2) \right]^{\frac{1}{2}} \hat{\rho}_{12} \\ \left[ p_1(1-p_1)p_2(1-p_2) \right]^{\frac{1}{2}} \hat{\rho}_{12} & \frac{p_2(1-p_2)}{n_{a2}} \left[ 1 + (n_{a2}-1)\hat{\rho}_{22} \right] \end{bmatrix}$$

TEH 0412762

ENAR Mtgs., Raleigh, NC, 3/25/85

May 11, 1985

DUP050453818

Table 1

Distribution of Favorable Response to Active Drug and Control  
Treatment From Multicenter Randomized Clinical Trial

| Clinic<br>No. | Treatment | Response  |             | Total | Proportion<br>Favorable |
|---------------|-----------|-----------|-------------|-------|-------------------------|
|               |           | Favorable | Unfavorable |       |                         |
| 1             | Drug      | 11        | 25          | 36    | 0.306                   |
|               | Control   | 10        | 27          | 37    | 0.270                   |
| 2             | Drug      | 16        | 4           | 20    | 0.800                   |
|               | Control   | 22        | 10          | 32    | 0.688                   |
| 3             | Drug      | 14        | 5           | 19    | 0.737                   |
|               | Control   | 7         | 12          | 19    | 0.368                   |
| 4             | Drug      | 2         | 14          | 16    | 0.125                   |
|               | Control   | 1         | 16          | 17    | 0.059                   |
| 5             | Drug      | 6         | 11          | 17    | 0.350                   |
|               | Control   | 0         | 12          | 12    | 0.000                   |
| 6             | Drug      | 1         | 10          | 11    | 0.091                   |
|               | Control   | 0         | 10          | 10    | 0.000                   |
| 7             | Drug      | 1         | 4           | 5     | 0.200                   |
|               | Control   | 1         | 8           | 9     | 0.110                   |
| 8             | Drug      | 4         | 2           | 6     | 0.667                   |
|               | Control   | 6         | 1           | 7     | 0.857                   |

May 11, 1985

ENAR Mtgs., Raleigh, NC, 3/25/85

TEH 0412763

DUP050453819

Table 2

Distribution of Favorable Response to Active Drug and  
Control Treatment Pooled Across Clinics 1 - 8

| Treatment | Distribution of Response |             | Total | Proportion<br>Favorable |
|-----------|--------------------------|-------------|-------|-------------------------|
|           | Favorable                | Unfavorable |       |                         |
| Drug      | 55                       | 75          | 130   | 0.423                   |
| Control   | 47                       | 96          | 143   | 0.329                   |
| Total     | 102                      | 171         | 273   | 0.374                   |

Table 3

Chi-square Statistics with  $df = 1$  and Fisher's Exact Test

| Criterion                       | Test<br>Statistic | Significance<br>Level |
|---------------------------------|-------------------|-----------------------|
| Pearson's(Uncorrected)          | 2.593             | 0.1073                |
| Yates'(Corrected)               | 2.206             | 0.1375                |
| Randomization Model             | 2.584             | 0.1080                |
| Likelihood Ratio Criterion      | 2.594             | 0.1073                |
| Fisher Exact Probability(Total) |                   | 0.13271               |
| Primary Tail                    |                   | 0.06875               |
| Secondary Tail                  |                   | 0.06396               |
| Observed Table                  |                   | 0.02746               |

TEH 0412764

ENAR Mtgs., Raleigh, NC, 3/25/85

May 11, 1985

DUP050453820

Table 4

Chi-square Statistics with  $df = 1$  Directed at Average Partial Association

| Criterion                  | Test Statistic | Significance Level |
|----------------------------|----------------|--------------------|
| Hypergeometric Model       |                |                    |
| Uncorrected(Birch)         | 6.384          | 0.01               |
| Corrected(Mantel-Haenszel) | 5.672          | 0.02               |
| Product Binomial Model     |                |                    |
| Uncorrected(Cochran)       | 6.559          | 0.01               |
| Corrected                  | 5.827          | 0.02               |
| Woolf WLS Test(Modified)   | 4.257          | 0.04               |

Table 5

Estimates of Variance Components Under Alternative Mixed Model Assumptions for Clinical Trial Data

| Mixed Effects Models          | Variance Components |                 |              |              |
|-------------------------------|---------------------|-----------------|--------------|--------------|
|                               | $\sigma_{e1}^2$     | $\sigma_{e2}^2$ | $\sigma_c^2$ | $\sigma_g^2$ |
| Interaction                   | 0.1901              | 0.1576          | 0.0700       | * 0.0019     |
| No Interaction/<br>No Pooling | 0.1901              | 0.1576          | 0.0710       | ...          |
| No Interaction/<br>Pooling    | 0.1866              | 0.1623          | 0.0709       | ...          |

TEH 0412765

May 11, 1985

ENAR Mtgs., Raleigh, NC, 3/25/85

Table 6

Estimates of Homogeneity and Cross-Homogeneity Coefficients for Clinical Trial Data

| Cluster Sampling Model | Homogeneity Coefficients |             |             |
|------------------------|--------------------------|-------------|-------------|
|                        | $\rho_{11}$              | $\rho_{22}$ | $\rho_{12}$ |
| Unconstrained          | 0.2160                   | 0.2750      | 0.2401      |

Table 7

Weighted Least Squares Analysis of Multicenter Randomized Clinical Trial Data

| Covariance Structure                       | Model-Based Test Statistics |      |                     |       |                    |
|--------------------------------------------|-----------------------------|------|---------------------|-------|--------------------|
|                                            | Lack of fit                 |      | Treatment(d.f. = 1) |       |                    |
|                                            | $Q_{LOF}$                   | d.f. | Significance Level  | $Q_C$ | Significance Level |
| Fixed Effects Model                        |                             |      |                     |       |                    |
| Unrestricted Multinomial                   | 8.76                        | 7    | 0.27                | 7.65  | 0.01               |
| Mixed Effects Model                        |                             |      |                     |       |                    |
| Mixed Effects (Interaction)                | 14.75                       | 14   | 0.40                | 5.35  | 0.02               |
| Mixed Effects (No Interaction/ No Pooling) | 15.99                       | 14   | 0.31                | 6.37  | 0.01               |
| Mixed Effects (No Interaction/ Pooling)    | 15.93                       | 14   | 0.32                | 6.34  | 0.01               |
| Cluster Sampling Model                     |                             |      |                     |       |                    |
| Variable Homogeneity Coefficients          | 16.21                       | 14   | 0.30                | 5.52  | 0.02               |

TEH 0412766

Table 11

Distribution of Elevated Diastolic Blood Pressure by Level of Blood Lead by  
NHANES II Sampling Site: Males, Ages 12-74, United States, 1976-80

| Sampling<br>Site No. | Blood Lead<br>$\mu\text{g/dl}$ | Diastolic Blood Pressure |           | Total | Proportion<br>$\geq 90$ |
|----------------------|--------------------------------|--------------------------|-----------|-------|-------------------------|
|                      |                                | $< 90$                   | $\geq 90$ |       |                         |
| 1                    | $< 15$                         | 16                       | 1         | 17    | 0.059                   |
|                      | $\geq 15$                      | 35                       | 10        | 45    | 0.222                   |
| 2                    | $< 15$                         | 21                       | 5         | 26    | 0.192                   |
|                      | $\geq 15$                      | 13                       | 6         | 19    | 0.316                   |
| 3                    | $< 15$                         | 3                        | 2         | 5     | 0.400                   |
|                      | $\geq 15$                      | 6                        | 1         | 7     | 0.143                   |
| 4                    | $< 15$                         | 6                        | 2         | 8     | 0.250                   |
|                      | $\geq 15$                      | 14                       | 2         | 16    | 0.125                   |
| ...                  |                                |                          |           |       |                         |
| ...                  |                                |                          |           |       |                         |
| 63                   | $< 15$                         | 32                       | 9         | 41    | 0.220                   |
|                      | $\geq 15$                      | 24                       | 8         | 32    | 0.250                   |
| 64                   | $< 15$                         | 42                       | 8         | 50    | 0.160                   |
|                      | $\geq 15$                      | 16                       | 5         | 21    | 0.238                   |

May 11, 1985

ENAR Mtgs., Raleigh, NC, 3/25/85

TEH 0412767

Table 12

Distribution of Elevated Diastolic Blood Pressure by Level of  
Blood Lead Pooled Across 64 NHANES II Sampling Sites:  
Males, Ages 12-74, United States, 1976-80

| Blood Lead<br>$\mu\text{g/dl}$ | Diastolic Blood Pressure |           | Total | Proportion<br>$\geq 90$ |
|--------------------------------|--------------------------|-----------|-------|-------------------------|
|                                | $< 90$                   | $\geq 90$ |       |                         |
| $< 15$                         | 1,134                    | 328       | 1,462 | 0.224                   |
| $\geq 15$                      | 1,222                    | 497       | 1,719 | 0.289                   |
| Total                          | 2,356                    | 825       | 3,181 | 0.259                   |

Table 13

Chi-square Statistics with  $df = 1$  for NHANES II  
Data Pooled Across 64 Sampling Sites

| Criterion                  | Test<br>Statistic | Significance<br>Level |
|----------------------------|-------------------|-----------------------|
| Pearson's(Uncorrected)     | 17.255            | $< 0.001$             |
| Yates'(Corrected)          | 16.920            | $< 0.001$             |
| Randomization Model        | 17.250            | $< 0.001$             |
| Likelihood Ratio Criterion | 17.365            | $< 0.001$             |

TEH 0412768

Table 14

Chi-square Statistics with  $df = 1$  Directed at Average Partial Association for NHANES II Data, Adjusting for 64 Sampling Sites

| Criterion                  | Test Statistic | Significance Level |
|----------------------------|----------------|--------------------|
| Hypergeometric Model       |                |                    |
| Uncorrected(Birch)         | 10.355         | <0.01              |
| Corrected(Mantel-Haenszel) | 10.073         | <0.01              |
| Product Binomial Model     |                |                    |
| Uncorrected(Cochran)       | 10.566         | <0.01              |
| Corrected                  | 10.279         | <0.01              |
| Woolf WLS Test(Modified)   | 7.341          | <0.01              |

Table 15

Estimates of Variance Components Under Alternative Mixed Model Assumptions for NHANES II Data

| Mixed Effects Models          | Variance Components |                 |              |              |
|-------------------------------|---------------------|-----------------|--------------|--------------|
|                               | $\sigma_{e1}^2$     | $\sigma_{e2}^2$ | $\sigma_c^2$ | $\sigma_g^2$ |
| Interaction                   | 0.1651              | 0.2013          | 0.0065       | 0.0001       |
| No Interaction/<br>No Pooling | 0.1651              | 0.2013          | 0.0066       | ...          |
| No Interaction/<br>Pooling    | 0.1672              | 0.1996          | 0.0066       | ...          |

May 11, 1985

ENAR Mtgs., Raleigh, NC, 3/25/85

TEH 0412769

DUP050453825

Table 16

Estimates of Homogeneity and Cross-Homogeneity Coefficients for NHANES II Data

| Cluster Sampling Model | Homogeneity Coefficients |             |             |
|------------------------|--------------------------|-------------|-------------|
|                        | $\rho_{11}$              | $\rho_{22}$ | $\rho_{12}$ |
| Unconstrained          | 0.056445                 | 0.020558    | 0.031679    |

Table 17

Weighted Least Squares Analysis of NHANES II Data

| Covariance Structure                       | Model-Based Test Statistics |      |                     |       |                    |
|--------------------------------------------|-----------------------------|------|---------------------|-------|--------------------|
|                                            | Lack of fit                 |      | Treatment(d.f. = 1) |       |                    |
|                                            | $Q_{LOF}$                   | d.f. | Significance Level  | $Q_C$ | Significance Level |
| Mixed Effects Model                        |                             |      |                     |       |                    |
| Mixed Effects (Interaction)                | 127.24                      | 126  | 0.45                | 13.29 | <0.001             |
| Mixed Effects (No Interaction/ No Pooling) | 128.36                      | 126  | 0.42                | 13.47 | <0.001             |
| Mixed Effects (No Interaction/ Pooling)    | 128.04                      | 126  | 0.43                | 13.43 | <0.001             |
| Cluster Sampling Model                     |                             |      |                     |       |                    |
| Variable Homogeneity Coefficients          | 121.43                      | 126  | 0.60                | 12.31 | <0.001             |

TEH 0412770

ENAR Mtgs., Raleigh, NC, 3/25/85

May 11, 1985

References

- Beitler, P.J. (1981). Mixed effects models for categorical response variables. Ph.D. dissertation, University of Michigan.
- Beitler, P.J. and Landis, J.R. (1985). A Mixed Effects Model for Categorical Data. Submitted to *Biometrics*.
- Kleinman, J.C. (1973). Proportions with extraneous variance: single and independent samples. *Journal of the American Statistical Association* 68, 46-54.
- Kleinman, J.C. (1975). Proportions with extraneous variance: two dependent samples. *Biometrics* 31, 737-743.
- Landis, J.R. and Koch, G.G. (1977). A one-way components of variance model for categorical data. *Biometrics* 33, 671-679.
- Lepkowski, J.M. (1980). Design effects for multivariate categorical interactions. Ph.D. dissertation, University of Michigan.

TEH 0412771

May 11, 1985

ENAR Mtgs., Raleigh, NC, 3/25/85

DUP050453827