

ficiently heavy to load the cork or felt support to the extent that the natural frequency of the machinery on the cork or felt is low in comparison with the frequency generated by the equipment, the cork or felt may be of little avail. The insulation of vibration can be accomplished by means of suitable elastic supports or suspensions, but the design of these elastic supports should be based upon calculation rather than guess-work.

The theory of the insulation of vibration was first worked out by Soderberg¹. If a machine of mass m be supported by an elastic pad the amount of vibratory force communicated by the machine to the floor or foundation upon which it rests will be determined by the elastic and viscous properties of the pad. The ratio of the vibratory force communicated to the floor or foundation with the machine resting upon the pad, and with the machine resting directly upon the floor, is given by the following equation:

$$\tau^1 = \sqrt{\frac{r^2 + \frac{1}{4\pi^2 n^2 c^2}}{r^2 + \left(2\pi n m - \frac{1}{2\pi n c}\right)^2}} \quad (4)$$

where

τ^1 = the so-called *transmissibility* of the support.

c = the compliance (that is, the reciprocal of the force constant).

r = the mechanical resistance owing to the viscous forces within the support.

n = the frequency of vibration generated by the machine which is to be insulated, such as the commutation frequency of a motor or the blade frequency of a fan.

m = the mass of the machine to be insulated.

It should be noted that not only must vibrations within the audible range of frequencies be considered, but those in the sub-audible range as well, since these may cause objectionable vibrations. All the possible frequencies should be considered in the calculation. Sometimes beat effects are introduced by slight irregularities of belts or pulleys that have much lower frequencies than those of the rotating elements.

If the pad is to be of any value in the prevention of solid-borne vibrations, the value of τ^1 must be considerably smaller than unity. If the fundamental frequency of vibration generated by the machine happens to coincide with the natural frequency of the mass of the machine resting on the elastic pad, a condition of resonance will be established, and the machine will exert a greater force upon the foundation than it would if the pad were completely removed. It is necessary, therefore, that the elastic support be sufficiently compliant, and the mass of the machine sufficiently heavy, that the natural frequency of the mass m upon its elastic support will be low in comparison with the frequencies which are generated by the machine. Thus, if the principal vibrations in the machine be of the order of 100 vibrations per second, the natural frequency of the machine mounted on its elastic support should not exceed about 20 vibrations per second.

If a slab of insulating material be placed under the entire foundation of a machine, as is often done in practice, it may happen that the natural frequency of the machine on its elastic support will be nearly the same as the frequencies which are to be insulated, in which case the elastic support

will be worse than nothing. In general, as Equation 4 shows, both m and c should be as large as possible if the vibrations of the machine are to be effectively insulated from the solid structure of the building. Furthermore, the machine should rest upon a rigid floor so that the elastic yielding of the floor is prevented from communicating the machinery vibrations to the solid structure of the building.

The elastic support under the machine acts as a low-pass filter which passes all frequencies below about two times the natural frequency of the machine mounted on its elastic support, but prevents all frequencies above about $\sqrt{\frac{mc}{\pi}}$ from reaching the solid structure of the building. The

principal influence of the internal mechanical resistance r is to limit the vibration at the resonant frequency. It is generally advisable, therefore, to use materials which have an appreciable internal resistance.

The values of c and r can be determined for any specimen of flexible material and, when known, can be used to determine the insulation value of any particular set-up. The value of c can be obtained by making static measurements of the amount of displacement of the compressed support for each additional unit of the compressing force. If this be done for a specimen of the flexible material of a certain thickness and area of cross section, the compliance can be determined for any other thickness or area from the relation that c will be directly proportional to the thickness and inversely proportional to the area of the flexible support. When the internal resistance r is not too large, it can be determined by observing the successive amplitudes of the free vibrations of a mass m which rests upon a specimen of the flexible material, and solving for r by the usual log-decrement method. Or, if the damping be so great that the free motion of m is non-oscillatory, r can be obtained from measurements on the experimentally-determined resonance curve of the forced vibrations of m , or from measurements of the rate of return of m when it is given an initial displacement.

If the resistance of a certain specimen of material, as cork, felt, or rubber, has been determined by any of these methods, the resistance for any other thickness or area of the material can be determined approximately because the resistance will be inversely proportional to the thickness and directly proportional to the area of cross-section of the flexible support. Thus, if the values of c and r for a flexible material be known, it is possible to calculate, by means of Equation 4, the amount of insulation that will be obtained from the use of this material as a flexible support for a piece of equipment having a mass m . For the routine calculations in practice, r may be neglected with only a slight sacrifice of accuracy. Table 4 gives the values of c and r for a number of commonly used flexible materials.

In general, there are two principal points to observe in the design of a flexible support for any piece of equipment, namely, the material should have a relatively large compliance and it should be loaded to nearly the upper safe limit of loading. Several flexible metallic supports have recently been developed.

Example 2. A machine weighing 1000 lb has a base area of 20 sq ft. Assume that the principal vibration of the machine has a frequency of 100 cycles per second (most

¹C. R. Soderberg, *The Electric Journal* (January, 1924), and succeeding articles. See also V. O. Knudsen, *Physical Review*, Vol. 32, 1928, p. 324, and A. L. Kimball, *Journal Acoustical Society of America*, Vol. 2, 1930, p. 297.