R_2, and into the water stream through the convection resistance, R_1. Note the analogy to the direct current electrical circuit problem. A temperature (potential) drop is required to overcome these resistances to the flow of thermal current. The total resistance to heat transfer, R_t, hour Fahrenheit degrees per Btu, is the summation of the individual resistances:

$$R_T = R_1 + R_2 + R_3 + R_4 \tag{9}$$

where the resultant parallel resistance R_4 is obtained from

$$\frac{1}{R_4} = \frac{1}{R_r} + \frac{1}{R_c} \tag{10}$$

Provided the individual resistances may be evaluated, the total resistance can be obtained from this relation. Then the heat transfer for the length of pipe (N, ft) can be established by the relation

$$q_{rc}\ (\text{Btu per hour}) = \frac{(t_o - t_l)}{R_T} \tag{11}$$

For a unit length of the pipe the heat transfer rate is

$$\frac{q_{rc}}{N}\ \text{Btu per (hour) (foot)} = \frac{(t_o - t_l)}{R_T N} \tag{12}$$

The temperature drop, Δt, through an individual resistance may then be calculated from the relation:

$$\Delta t = R\, q_{rc} \tag{13}$$

where R is the resistance in question.

The problem is now reduced to one of evaluating the individual resistances of the system. This entails suitable manipulation of the rate Equations 1, 2 and 3 to produce expressions of the form:

$$q = \frac{\Delta t}{R} \tag{14}$$

where q is the heat transfer rate, and Δt is the potential drop or temperature difference through the resistance R. Table 6 lists such solutions for six different conduction systems. Table 2 in Chapter 9 and Table 1 of this chapter indicate the magnitudes of the thermal conductivities, k, to be employed in the expressions of Table 6, after dividing k by 12.

The solution applicable to the problem depicted in Fig. 8, for the calculation of R_2 and R_3, is case 2 in Table 6. Thus for a *1 ft length* of 2 in. nominal size pipe (I. D. = 2.067 in., O. D. = 2.375 in.) insulated with 1 in. of material having a conductivity of 0.025:

$$R_2 = \frac{\log_e \frac{1.188}{1.033}}{2\pi \times 26 \times 1} = 8.5 \times 10^{-4}\ \text{(hr) (F deg) per Btu.}$$

$$R_3 = \frac{\log_e \frac{2.188}{1.188}}{2\pi \times 0.025 \times 1} = 3.9\ \text{(hr) (F deg) per Btu.}$$

The convection resistances to heat transfer from the pipe wall to the cold water, R_1, and from the air to the surface of the insulating material, R_c, are dependent on the flow conditions prevailing at these surfaces, and on the thermal properties of the fluids. These resistances are also directly dependent upon the temperature distribution, and for this reason it is necessary first, to guess on the basis of the problem statement, a temperature distribution upon which to base the initial calculations. Since the values of h_c for heat transfer between water and pipe walls are relatively high in this temperature range, it is logical to assume only a small temperature difference between the temperature of the fluid body and the temperature of the pipe wall. For the purpose of an initial guess, this temperature difference will be assumed to be 2 deg. On the other hand, h_c for heat transfer from air to a body is relatively small, and a higher temperature difference would be expected between these masses. The value initially assumed here will be 20 deg. In summary, the temperature distribution in the system is assumed as follows:

Fluid temperature = 34 F.
Inner pipe wall temperature = 36 F.
Outer insulation surface temperature = 100 F.
Ambient air temperature = 120 F.

With these assumptions and the problem statement, it is now possible to calculate values for the convective resistances. If it is found in the ultimate solution of the problem that the temperature distribution is different from that assumed, it will then be necessary to repeat the solution procedure.

If reference now be made to Table 2, it is found that Case 3 of this table is a system similar to that encountered in the convection between the water and the pipe wall. The equation for this case is the following:

$$h_c = 13.9(t_l)^{0.548} \frac{u_m^{0.8}}{D^{0.2}} \tag{15}$$

where

$$u_m = 5\ \text{fps}$$

$$D = \frac{2.067}{12} = 0.1725\ \text{ft}$$

$$t_l = \frac{34 + 36}{2} = 35\text{F}$$

$$h_c = \frac{13.9 \times 6.9 \times 3.62}{0.703} = 494\ \text{Btu per (hr) (sq ft) (F deg).}$$

This heat transfer rate is through the inner surface of the pipe and it is, therefore, this area that determines the resistance R_1.

$$A = \pi D = 0.542\ \text{sq ft per unit length of pipe,}$$

and therefore

$$R_1 = \frac{1}{h_c A} = \frac{1}{494 \times 0.542}$$

$$R_1 = 3.73 \times 10^{-3}\ \text{(hr) (F deg) per Btu.}$$