= 31,600, only 1.9 percent. Consequently, the values of $f/2$ at high Reynolds number, obtained by difference, are subject to considerable error.

In Fig. 10 some data on the evaporation of water into air for single spheres are presented. The solid line, which best represents these data, is in quite good agreement with the dashed line representing McAdams correlation for heat transfer to spheres. It is not possible to compare these results with friction or momentum transfer, since the breakdown of total drag into skin friction and normal pressure drag has not been made. The application of these data to air-water contacting devices such as air washers and spray cooling towers is well substantiated.

In summary, the equality of j_H, j_D, and $f/2$ for certain streamline shapes at low mass transfer rates has excellent experimental verification. For flow past bluff objects, j_H and j_D are much smaller than $f/2$, based on total pressure drag. The heat and mass transfer are, however, still related in a useful way by equating j_H and j_D.

Example 2. Using solid cylinders of volatile solids (e.g., naphthalene, camphor, dichlorobenzene) with air flow normal to these cylinders, Bedingfield and Drew[9] found that the ratio between the heat and mass transfer coefficients could be closely correlated by the relation

$$h/\rho h_D = 0.294(\mu/\rho D_v)^{0.56}$$

For completely dry air at 70 F flowing at a velocity of 31 fps over a wet-bulb thermometer of diameter d = 0.300 in., determine the heat and mass transfer coefficients from Fig. 9 and compare their ratio with the Bedingfield-Drew relation.

Solution: For dry air at 70 F, $\rho = 0.075$ lb per cu ft, $\mu = 0.0441$ lb per (ft) (hr), $k = 0.0150$ Btu per (hr) (ft²) (F deg per ft), $c_p = 0.2398$ Btu per (lb) (F deg). From Equation 7, $D_v = 0.971$ sq ft per hr. Therefore

$$N_{Re} = \rho V d/\mu = 0.075(31)(3600)(0.300)/12(0.0441) = 475$$

$$N_{Pr} = c_p\mu/k = (0.2398)(0.0441)/0.0150 = 0.706$$

$$N_{Sc} = \mu/\rho D_v = 0.0441/0.075(0.971) = 0.605$$

From Fig. 9 at $N_{Re} = 475$, $j_H = j_D = 0.025$ and

$$h = j_H c_p V_m/(N_{Pr})^{2/3}$$

$$h = (0.025)(0.075)(0.2398)(31)(3600)/(0.706)^{2/3}$$

$$= 63.3 \text{ Btu per (hr)(sq ft)(F deg)}$$

$$h_D = j_D V_m/(N_{Sc})^{2/3} = (0.025)(31)(3600)/(0.605)^{2/3}$$

$$= 3900 \text{ ft per hr}$$

$$h/\rho h_D = 63.3/3900(0.075) = 0.216 \text{ Btu per (lb)(F deg)}$$

From the Bedingfield and Drew relation

$$h/\rho h_D = 0.294(0.605)^{0.56} = 0.222 \text{ Btu per (lb)(F deg)}$$

Note further that the Reynolds analogy, Equation 24, suggests that $h/\rho h_D = c_p = 0.2398$ Btu per (lb) (F deg). The close agreement here is due to the fact that the ratio N_{Sc}/N_{Pr} is 0.605/0.706 or 0.85, so that the exponent of these numbers has little effect on the ratio of the transfer coefficients.

The Lewis Relation

The experimental verification of the similarity relations presented in the preceding section suggests that the heat and mass transfer coefficients are satisfactorily related, at the same Reynolds number, by equating the Chilton-Colburn j-factors. This leads to

$$\frac{h}{V_m P_t}\left(\frac{\mu}{\rho D_v}\right)^{2/3} = \frac{h}{\rho c_p V_m}\left(\frac{c_p\mu}{k}\right)^{2/3}$$

or

$$\frac{h}{h_D \rho c_p} = \frac{p_{am}}{P_t}\left[\frac{\mu/\rho D_v}{c_p\mu/k}\right]^{2/3} = \frac{p_{am}}{P_t}\left(\frac{\alpha}{D_v}\right)^{2/3} \quad (29)$$

The quantity α/D_v is called the Lewis number N_{Le}. Its magnitude expresses relative rates of propagation of energy and mass within a system. It is fairly insensitive to temperature variation. For air and water vapor mixtures the ratio is (0.60/0.71) or 0.845, and $(0.845)^{2/3}$ is 0.894. Furthermore, at low diffusion rates, where the heat-mass transfer analogy is valid, the ratio (p_{am}/P_t) is essentially unity. Therefore, for air and water vapor mixtures, we can set

$$\frac{h}{h_D \rho c_p} = 1 \quad (30)$$

That is, the ratio of the heat transfer coefficient to the mass transfer coefficient is equal to the specific heat per unit volume at constant pressure of the mixture. This relation, first derived by W. K. Lewis,[10] is usually called the Lewis relation.

It is important to recognize that this relation is fortuitously very nearly true for air and water vapor at low mass transfer rates. It is not, in general, true for other gas mixtures because the ratio of thermal to vapor diffusivity, N_{Le}, is usually different from unity. Another interesting point which deserves attention here is that the reading of a wet-bulb thermometer owes its importance to the validity of the Lewis relation. That is, agreement between wet-bulb temperature and adiabatic saturation temperature is a direct result of the nearness of Lewis number to unity for air and water vapor.

The Lewis relation is valid in turbulent flow whether or not the ratio of α/D_v equals 1. This follows from the fact that the eddy diffusion process in turbulent flow involves the same macroscopic mixing action for heat exchange as for mass exchange, and this action completely overwhelms any molecular diffusion process. The deviations from the Lewis relation are, therefore, due to the presence of a laminar boundary layer as in Fig. 2, or a laminar sublayer and buffer zone as in Fig. 6 wherein molecular transport phenomena are the controlling factor.

SIMULTANEOUS HEAT AND MASS TRANSFER BETWEEN WATER-WETTED SURFACES AND AIR

As stated in the section Fundamental Principles, a simplified method is commonly used for solving problems involving simultaneous heat and mass transfer. This method, which is developed by use of the Lewis relation, gives satisfactory results for most air-conditioning processes. It should be noted, however, that extrapolation to very high mass transfer rates, where the simple heat-mass transfer analogy is not valid, probably will lead to erroneous results.

Enthalpy Potential

The quantity usually used to express the water vapor concentration in the air is the humidity ratio W, defined as

$$W = m_v/m_a = \rho_v/\rho_a$$

It is, therefore, convenient to define a mass transfer coefficient using W as the driving potential, hence

$$\dot{m}_v/A = K_D(W_i - W_m) \quad (31)$$

where the coefficient K_D has the units pounds per (hour) (square foot). Using the definition of W and noting that for dilute mixtures $\rho_{ai} \cong \rho_{am}$ (i.e., the partial mass density of dry air changes only by a small percentage between interface and free stream conditions), it is permissible to write

$$\dot{m}_v/A = \frac{K_D}{\rho_{am}}(\rho_{vi} - \rho_{vm})$$

where ρ_{am} is the *mean* density of dry air, pounds per cubic foot. Comparing this equation with Equation 2a shows that

$$h_D = K_D/\rho_{am}$$

The *humid* specific heat c_{pm} of the air stream is, by definition

$$c_{pm} = (1 + W_m)c_p \text{ Btu per (F deg)(lb of dry air)}$$

or

$$c_{pm} = (\rho/\rho_{am})c_p$$

Substituting from these latter expressions into the Lewis relation (Equation 30) gives

$$\frac{h\rho_{am}}{K_D\rho_{am}c_{pm}} = 1 \cong \frac{h}{K_D c_{pm}} \quad (32)$$

since $\rho_{am} \cong \rho_{am}$ due to the small change in dry-air density. Using a mass transfer coefficient with humidity ratio as the driving force, the Lewis relation becomes: ratio of heat to mass transfer coefficient equals humid specific heat.

For the pan humidifier illustrated in Fig. 1, it has been shown that the total heat transfer from the liquid to the interface is

$$q/A = q_v/A + (\dot{m}_v/A)h_{fv}$$

Utilising the definitions of the transfer coefficients,

$$q/A = h(t_i - t_m) + K_D(W_i - W_m)h_{fv}$$

Assuming the Lewis relation (Equation 32)to be valid gives

$$q/A = K_D[c_{pm}(t_i - t_m) + (W_i - W_m)h_{fv}] \quad (33)$$

The enthalpy of the air is, by definition

$$h = c_{pa}t + Wh_v \text{ (Btu per lb dry air)}$$

where

h = enthalpy of air, Btu per pound.
h_v = enthalpy of water vapor, Btu per pound.

The enthalpy of the water vapor h_v may, by the perfect gas law be expressed as

$$h_v = c_{pv}(t - t_o) + h_{fo}$$

where the base of enthalpy is taken as saturated water at temperature t_o. Choosing $t_o = 0$ F to correspond with the base of the dry-air enthalpy gives

$$h = (c_{pa} + Wc_{pv})t + Wh_{fo} \cong c_{pa}t + Wh_{fo} \quad (34)$$

Comparing Equations 33 and 34, it is obvious that if small changes in the latent heat of vaporization of water with temperature are neglected, the total heat transfer can be written

$$q/A = K_D(h_i - h_m) \quad (35)$$

Thus, whereas the driving potential for heat transfer is temperature difference and the driving potential for mass transfer is mass concentration or partial pressure, *the driving potential for simultaneous transfer of heat and mass in an air water-vapor mixture is enthalpy.*

Basic Equations for Direct-Contact Equipment

Air-conditioning equipment may be classified according to whether there is direct contact between water used as a cooling or heating fluid and air, or whether the heating or cooling fluid is separated from the air stream by a solid wall. Examples of the former are air washers and cooling towers, whereas the most common example of the latter is a direct expansion refrigerant (or water) cooling and dehumidifying coil. It should be noted that in both cases the air stream is in contact with a water surface. The term *direct contact* implies that the contact is directly with the cooling (or heating) fluid. In the dehumidifying coil the contact is directly with the condensate removed from the air stream, but indirectly with the refrigerant flowing inside the tubes of the coil. The inability to evaluate surface areas in direct-contact equipment is the principal reason for treating these two cases separately.

For the direct-contact spray chamber of cross-sectional

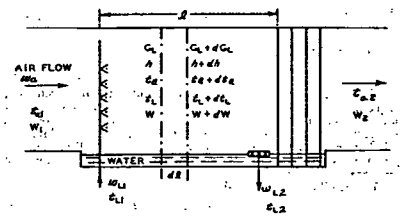

Fig. 11 . . . . Air Washer Spray Chamber

area A_{ca} and length l (Fig. 11), the steady mass flow rate of dry air is

$$w_a/A_{ca} = G_a$$

and the mass flow rate of water flowing parallel with the air is

$$w_L/A_{ca} = G_L$$

where

w_a = mass flow rate of air, pounds per hour.
G_a = mass velocity, or flow rate per unit cross-sectional area, for air, pounds per (hour) (square foot).
w_L = mass flow rate of liquid, pounds per hour.
G_L = mass velocity, or flow rate per unit cross-sectional area, for liquid, pounds per (hour) (square foot).

Since water is evaporating or condensing, G_L changes by an amount dG_L in a differential length dl of the chamber. Similar changes occur in temperature, humidity ratio, enthalpy, and other properties.

Because it is difficult to evaluate true surface area in direct contact equipment; common practice is to work on a unit volume basis. If a_H and a_M represent the square feet of heat transfer and mass transfer surface per cubic foot of chamber volume, respectively, the total surface areas for heat and mass transfer are then

$$A_H = a_H A_{ca} l \text{ and } A_M = a_M A_{ca} l$$

The basic equations for the process occurring in the differential length dl may be written for:

(1) Mass Transfer

$$-dG_L = G_a dW = K_D a_M(W_i - W)dl \quad (36)$$

That is, the water evaporated, the moisture increase of the air, and the mass transfer rate are all equal.

(2) Heat Transfer to the Air

$$G_a c_{pm} dt_a = h_a a_H(t_i - t_a)dl \quad (37)$$

(3) Total Energy Transfer to the Air

$$G_a(c_{pm}dt_a + h_{fo}dW) = [K_D a_M(W_i - W)h_{fv} + h_a a_H(t_i - t_a)]dl \quad (38a)$$

As shown in the preceding section, assuming $a_H = a_M$, $N_{Le} = 1$, and neglecting small variations in h_{fv} Equation 38a reduces to

$$G_a dh = K_D a_M(h_i - h)dl \quad (38b)$$

Note: The assumption that the heat and mass transfer areas are identical ($a_H = a_M$) is usually quite valid for spray chambers. In equipment where packing materials (e.g., wood slats, Raschig rings, etc.) are used, however, there may be considerable difference in the two areas due to nonuniform wetting of the packing. The validity of the Lewis relation has been adequately discussed previously. To attempt to account for the very small changes in