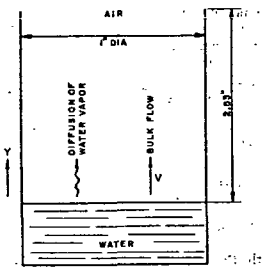

Fig. 4 .... Diffusion of Water Vapor Through *Stagnant* Air

both components behave as perfect gases and that the Gibbs-Dalton law is valid, hence

$$p_a + p_v = P_t = \text{constant} \tag{11}$$

where

P_t = total pressure, pounds force per square foot = 2116 P_T

Thus, the partial pressure of the air p_a must decrease as the partial pressure of the water vapor p_v increases, so that the total pressure P_t is constant. Dividing both sides of Equation 5c by M_v, the molecular weight of water,

$$\dot{n}_v/A = -(D_v/R_vT)(dp_v/dy) \tag{12}$$

Similarly

$$\dot{n}_a/A = -(D_v/R_vT)(dp_a/dy) \tag{13}$$

where

$\dot{n}_v$ = molal diffusion rate of water vapor, pound-mols per hour,

R_v = universal gas constant = 1545 foot-pounds force per (Rankine degree) (pound-mol).

$\dot{n}_a$ = molal diffusion rate of dry air, pound-mols per hour.

y = distance measured normal to the transfer surface, feet.

D_v is the same for diffusion of water vapor through air as for air through water vapor. Equation 11 requires that $(dp_v/dy) = -(dp_a/dy)$. By integration between planes 1 and 2 of Fig. 3,

$$\dot{n}_v/A = -\dot{n}_a/A = \frac{-D_v(p_{v2} - p_{v1})}{R_vT(y_2 - y_1)} \tag{14}$$

This process is known as *equimolal counterdiffusion*. The rates of diffusion of air and water vapor are equal, but in opposite directions. This situation has no counterpart in heat transfer, since heat can be transferred only in one direction at a time.

Diffusion of One Gas Through a Second Stagnant Gas

A different type of diffusion process is that in which one gas diffuses through a second stationary or *stagnant* gas. An important example of this type of diffusion, illustrated in Fig. 4, is where water vapor diffuses from the liquid surface into surrounding stationary air. It is assumed that equilibrium exists through the gas mixture, that the gases are perfect, and that the Gibbs-Dalton law is valid. As usual, the diffusion of the water vapor is due to a concentration or partial pressure gradient, and is given by Equation 12.

There is a continuous gas phase, so the total pressure, P_t, is constant, and Equation 11 is valid. The partial pressure gradient of the water vapor causes a partial pressure gradient of the air such that, $dp_a/dy = -dp_v/dy$. Air, then, diffuses toward the liquid water interface. Since it cannot be absorbed there, a convective or bulk velocity v of the gas mixture is established in a direction away from the liquid surface, so that the net transport of air is zero; i.e., it is *stagnant*.

$$\dot{n}_a/A = -\frac{D_v}{R_vT}\frac{dp_a}{dy} + \bar{\rho}_a v = 0$$

and

$$v = \frac{D_v}{R_vT}\frac{1}{\bar{\rho}_a}\frac{dp_a}{dy} = -\frac{D_v}{R_vT}\frac{1}{\bar{\rho}_a}\frac{dp_v}{dy} \tag{15}$$

where $\bar{\rho}_a$ is the molal density of the air (in lb mols per cu ft).

The bulk velocity, v, not only gives rise to a transport of air, but also of water vapor away from the interface. The total rate of water vapor diffusion is therefore

$$\dot{n}_v/A = -\frac{D_v}{R_vT}\frac{dp_v}{dy} + \bar{\rho}_v v$$

Substituting for the velocity v from Equation 15 gives

$$\dot{n}_v/A = -\frac{D_v}{R_vT}\left(1 + \frac{\bar{\rho}_v}{\bar{\rho}_a}\right)\frac{dp_v}{dy} = -\frac{D_v}{R_vT}\frac{\bar{\rho}}{\bar{\rho}_a}\frac{dp_v}{dy}$$

But the molal density ratio $\bar{\rho}/\bar{\rho}_a = P_t/p_a$ from the ideal gas mixture relations, so

$$\dot{n}_v/A = -\frac{D_v}{R_vT}\frac{P_t}{p_a}\frac{dp_v}{dy} = \frac{D_v}{R_vT}\frac{P_t}{p_a}\frac{dp_a}{dy} \tag{16}$$

Integration yields

$$\dot{n}_v/A = \frac{D_vP_t}{R_vT}\frac{\log_e(p_{a2}/p_{a1})}{(y_2 - y_1)} = -\frac{D_v}{R_vT}\frac{P_t}{p_{am}}\frac{(p_{a2} - p_{a1})}{(y_2 - y_1)} \tag{17}$$

where p_{am} is the logarithmic mean partial pressure of the *stagnant* air. The partial pressure gradients for this type of diffusion are illustrated in Fig. 5.

It should be noted that the term *stagnant* refers to the net behavior of the air. The air is moving, but the convective flow exactly offsets the diffusion. Comparison of Equation 17 with Equation 14 shows that the factor P_t/p_{am} is introduced when diffusion through a stagnant gas is considered. For a dilute mixture of water vapor in air, p_{am} is approximately equal to P_t and Equations 14 and 17 become identical.

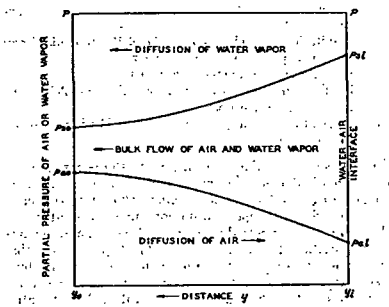

Fig. 5 .... Partial Pressure Profiles for Diffusion of Water Vapor Through *Stagnant* Air

Example 1. A vertical tube of 1.000 in. diameter is partially filled with water so that the distance from the water surface to the open end of the tube is 2.530 in. (Fig. 4). Perfectly dried air is blown over the open tube end and the complete system is at a constant temperature of 85 F. The water evaporated from the tube is found to be 0.0102 lb in 200 hr of steady operation. The total pressure of the system is 14.696 psia.

a) Using these data, calculate the mass diffusivity of water vapor in air.

b) Compare this experimental result with that calculated by Equation 7.

Solution: (a) The molal diffusion rate $\dot{n}_v$ is

$$\frac{0.0102}{200 \times 18} = 0.00000283.$$

The area of a 1 in. diameter tube is 0.00545 sq ft. Therefore, $(\dot{n}_v/A)_1 = 0.00000283/0.00545 = 0.00052$ lb-mols per (hr)(sq ft). The partial pressures are:

$$p_{v2} = 0; \quad p_{v1} = 0.596 \text{ psi}$$

$$p_{a1} = P_t - p_{v1} = 14.696 - 0.596 = 14.10 \text{ psia}$$

The logarithmic mean partial pressure of the air is

$$p_{am} = \frac{p_{a2} - p_{a1}}{\log_e(p_{a2}/p_{a1})} = \frac{0.596}{\log_e(14.696/14.10)} = 14.38 \text{ psia}$$

The mass diffusivity is now computed from Equation 17 as

$$D_v = \frac{(\dot{n}_v/A)R_vTp_{am}(y_2 - y_1)}{P_t(p_{v1} - p_{v2})}$$

$$D_v = \frac{(0.00052)(1545)(545)(14.38)(2.53/12)}{(14.696)(0.596)(144)}$$

$$= 1.050 \text{ sq ft per hr}$$

b) By Equation 7, noting that $P_T = P_t/14.696 = 1.00$ atmospheres,

$$D_v = \frac{0.000146}{P_T}\frac{T^{2.5}}{(T + 441)}$$

$$D_v = 0.000146(545)^{2.5}/986 = 1.025 \text{ sq ft per hr}$$

Note that neglecting the correction factor P_t/p_{am} for this example would cause only a 2 percent change in the calculated experimental value of D_v.

CONVECTIVE OR EDDY DIFFUSION

The concept of turbulent flow is described by the characteristic phenomenon of random velocity fluctuations superimposed on the time-average velocity. These velocity fluctuations result in small particles of the fluid moving, at one instant, faster than the average velocity; at another instant, slower. The fluctuations occur not only in the direction of flow but also normal to the flow direction. As a result, small mixing actions or *eddy currents* are established within the turbulent flow field. If a small particle of fluid is carried by an eddy current from a region of high velocity to one of low velocity, it will give up momentum to the slower moving fluid. Hence, these eddies result in an exchange of momentum between different layers of the moving fluid. If there is a temperature gradient in the fluid, energy will be exchanged by this same mixing action of the eddies in exactly the same way. Similarly, if a mass concentration gradient exists, there will be a mass exchange by the same mechanism. This mass exchange is known as *eddy diffusion*. The difference between molecular and eddy diffusion is that of microscopic or molecular mixing action versus macroscopic or particle mixing action. As might be expected, eddy diffusion is relatively fast as compared to molecular diffusion. Furthermore, the rate of eddy diffusion will depend on the *intensity* of the velocity fluctuations; that is, on the intensity of the turbulence. Since the intensity of turbulence is determined by the Reynolds number of the flow, the rate of eddy diffusion will depend on the Reynolds number. It is possible to define an *eddy diffusivity* ϵ_D by the same form of equation as that in molecular diffusion (i.e., Equation 8), so that

$$\dot{m}_v/A = -\epsilon_D(d\rho_v/dy) \tag{18}$$

where

ϵ_D = eddy diffusivity, square feet per hour.

Experiments have shown that ϵ_D is a strong function of Reynolds number.[2] Because data on eddy diffusivities are rare and difficult to obtain common practice is to define a mass transfer coefficient, analogous to the heat transfer coefficient in convective heat transfer, and to determine it experimentally.

Mass Transfer Coefficient

Consider an air stream in steady turbulent flow over a wetted surface as illustrated in Fig. 6. It is assumed that the liquid-vapor interface is stationary and, therefore, the velocity is zero at this surface. This results in a slow-moving layer of fluid next to the surface which is in laminar flow. Between this *laminar sublayer* and the main body of the turbulent stream there is a transition region in which the fluid may be alternately in laminar flow and in turbulent flow. This flow regime is referred to as the *buffer layer*. Within the laminar sublayer, only molecular diffusion can occur; whereas in the buffer layer both molecular and eddy diffusion are important contributors to the mass transfer process. In the turbulent region eddy diffusion predominates and is so rapid as to soon equalize the concentration gradient.

Because of the presence of the laminar sublayer, the rate of molecular mass diffusion from the wetted surface to the air stream, from Equation 5c is

$$\text{Diffusion rate} = -(D_v/R_vT)(dp_v/dy)_i$$

where $(dp_v/dy)_i$ is the partial pressure gradient at the interface. Making the usual assumptions that the gases are ideal, that they obey the Gibbs-Dalton law, and that the total pressure is constant, then a partial pressure gradient must also exist in the air. The wetted surface is impermeable to air. Therefore, a convective or bulk velocity must be established to counter the air diffusion rate. The total mass transfer from the wetted surface to the air stream must then be given by

$$\dot{m}_v/A = -(D_v/R_vT)(dp_v/dy)_i + \rho_{vi}v_i \tag{19}$$

where v_i is the convective velocity at the interface (Fig. 6), and $\rho_{vi} = p_{vi}/R_vT$ is the partial mass density of the water vapor at the interface.

It was shown previously that for the diffusion of one gas through a second stagnant gas, the simple molecular diffusion equation could be properly corrected by the factor (P_t/p_{am}), thus accounting for the mass transfer contribution of the convective velocity. Furthermore, in the case of dilute mixtures, as in *Example* 1 above, this contribution is small. It might be assumed, therefore, that the same correction factor could be

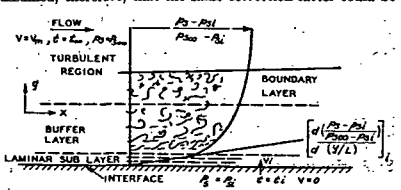

Fig. 6 .... Turbulent Diffusion Boundary Layer on a Flat Surface