solid and partly broken) of Fig. 6 is found. The maximum value of $\frac{p_2}{p_1}$ may be computed by differentiating w with respect to p_2 and equating the result to zero. This operation produces the formula:

$$\frac{p_2}{p_1} = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}} \tag{40}$$

For air, with $k = 1.40$, $\frac{p_2}{p_1} = 0.53$.

Actually, the broken part of the curve is not attained for the flow in the nozzle. If the ratio of p_2 to p_1 is decreased from unity, the mass rate of discharge, as well as the volume, increases from zero to a maximum, as shown by the solid section of the curve in Fig. 6; thereafter, as p_2/p_1 is decreased further, the discharge is constant, as indicated by the horizontal line. The value of p_2 at the maximum point is called the critical pressure, or p_c, and it is seen that p_c is approximately 53 percent of p_1 when air is flowing.

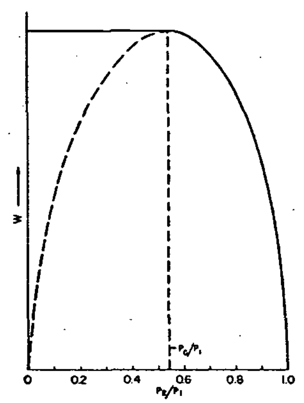

FIG. 6. RELATION OF FLOW OF GAS TO PRESSURE DROP IN A CONVERGING TUBE

To find the velocity at the critical pressure, it is assumed that the upstream velocity V_1 is so small as to be negligible. Using the subscript c to indicate conditions at the critical point, from Equation 29

$$\frac{p_c}{p_1} = \frac{1}{\left(1 + \frac{k-1}{2} M_c^2\right)^{\frac{k}{k-1}}} \tag{41}$$

or

$$M_c = \sqrt{\frac{2}{k-1}\left[\left(\frac{p_1}{p_c}\right)^{\frac{k-1}{k}} - 1\right]} \tag{42}$$

Substituting the critical pressure ratio from Equation 40 it follows that

$$M_c = 1 \tag{43}$$

or that the velocity at the throat is equal to the local sonic velocity.

In developing the working equations for orifices and nozzles, it is customary to start with the incompressible form of the flow Equation 38. In this case both M_1 and M_2 are small quantities, $\rho_1 = \rho_2$, and $(p_1 - p_2)/p_2 = \Delta p/p_2$ is small. Retaining only first order terms, it follows from Equation 35 that

$$\frac{M_1}{M_2} = \frac{A_2}{A_1} \tag{44}$$

so that

$$\sqrt{\frac{1 + \frac{k-1}{2} M_1^2}{1 - M_1^2/M_2^2}} \cong \frac{1}{\sqrt{1 - (A_2/A_1)^2}} = \frac{1}{\sqrt{1-\beta^4}} \tag{45}$$

where $\beta = D_2/D_1$. The quantity $1/\sqrt{1-\beta^4}$ is the velocity of approach

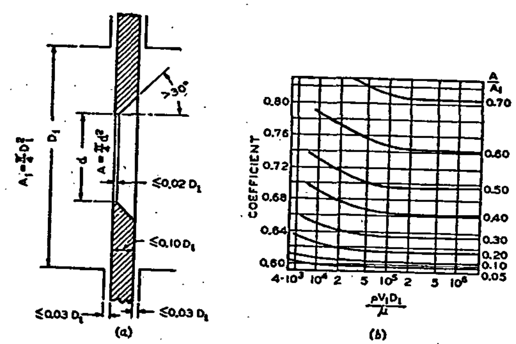

FIG. 7. DIMENSIONS AND FLOW COEFFICIENT FOR STANDARD SHARP EDGED ORIFICE (Coefficient shown as a function of Reynolds Number and Ratio, A/A.) NOTE: From Reference 4. Used by permission.

factor as generally used, with β being the ratio of the throat or orifice diameter to the pipe diameter.

Since $\Delta p/p_2$ is small

$$\frac{2k}{k-1}\left[\left(\frac{p_1}{p_2}\right)^{\frac{k-1}{k}} - 1\right] \cong \frac{2\Delta p}{p_2} \tag{46}$$

and the mass flow is

$$w = \frac{A_2}{\sqrt{1-\beta^4}} \sqrt{2g\rho\Delta p} \tag{47}$$

The volume flow is then

$$Q = A_2 \frac{1}{\sqrt{1-\beta^4}} \sqrt{2g\Delta p/\rho} = A_2 \frac{1}{\sqrt{1-\beta^4}} \sqrt{2gh_t} \tag{48}$$