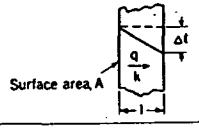
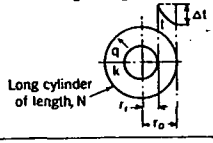
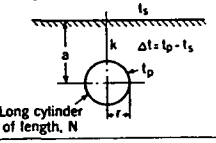
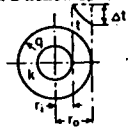
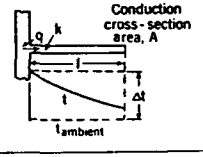
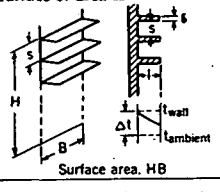
TABLE 6. SOLUTIONS FOR SOME STEADY-STATE THERMAL CONDUCTION PROBLEMS[ab]

No.	System	Expressions for the resistance R entering into the equation: $q = \Delta t/R$ (Btu per hour)
1.	Flat wall or curved wall if curvature is small (wall thickness less than 0.1 of inside diameter). 	$R = \frac{l}{kA}$
2.	Radial flow through a right circular cylinder. 	$R = \frac{\log_e \frac{r_o}{r_i}}{2\pi kN}$ (See footnote *c*).
3.	The buried cylinder. 	$R \approx \frac{\log_e \frac{2a}{r}}{2\pi kN}$; $R = \frac{\cosh^{-1}\frac{a}{r}}{2\pi kN}$ for $\frac{a}{r} \geq 3$ (See footnote *c*).
4.	Radial flow in a hollow sphere. 	$R = \frac{\frac{1}{r_i} - \frac{1}{r_o}}{4\pi k}$
5.	The straight fin or rod heated at one end. 	$R = \frac{m}{h_s p \tanh ml}$ (see footnotes *d* and *e*). For $ml > 2.3$, $\tanh m\,l \approx 1$ $m = \sqrt{h_s p/kA}$ A = conduction cross section area. p = perimeter of cross section A. h_s = unit conductance to the surroundings from the fin surface. k = thermal conductivity fin material. Δt = wall temperature—ambient temp.
6.	Finned surface of area HB.  Surface area, HB	$R = \frac{(s+\delta)}{h_s\left(\frac{2}{m}\tanh m\,l + s\right)HB}$ $m = \sqrt{\frac{h_s p}{kA}} = \sqrt{\frac{2\,h_s}{k\delta}}$ Δt defined as in Case 5 above.

[a] The dimensions to be employed in these solutions are: length of dimension p, l, r = feet; units of k = Btu per hour per square foot per degree Fahrenheit for one foot thickness; units of h, Btu per hour per square foot per degree Fahrenheit; units of area, A = square feet.

[b] The thermal conductivity, k, in these solutions should be taken at the average material temperature (see Table 7).

[c] $\text{Log}_e\, x = 2.303 \log_{10} x$.

[d] This expression can also be employed as an approximation for tapered fins or of annular fins by employing average magnitudes of A and p.

[e] *Tanh is the hyperbolic tangent.*

G = 5 (ft per sec) × 62.4 (lbs per cu ft) × 3600 (sec per hr) = 11.2×10^5 lb per hour per square foot.

The hydraulic radius for the flow in the 2 in. line is, by definition:

$$r_H = \frac{\pi D^2}{\pi D 4} = \frac{D}{4} = \frac{2.067}{12 \times 4} = \frac{0.1725}{4} \text{ ft.}$$

The average water film temperature will be estimated as 36 F (mixed mean fluid temperature of 34 F). Then case 3, Table 7 yields:

$$h = 0.00486\,(1 + 0.36)\,\frac{(11.2 \times 10^5)^{0.8}}{(0.1725)^{0.2}} = 650 \text{ Btu per hour per square foot per degree Fahrenheit.}$$

The transfer area on which this conductance is based is the inside tube area. Associated with 1 ft length of pipe there are:

$$\pi \times \frac{2.067}{12} \times 1 = 0.542 \text{ sq ft.}$$

Thus the resistance for 1 ft of tube length is:

$$R_1 = \frac{1}{h\pi D \times 1} = \frac{1}{650 \times 0.542} = 2.8 \times 10^{-3} \text{ hr degree Fahrenheit per Btu.}$$

Case 9, Table 7 is applicable for calculating the free thermal convection resistance, R_c, existing between the surrounding air and the insulation. The air temperature is given as 120 F. As an approximation a 20 F temperature difference between the air and the pipe surface will be assumed. Then case 9 yields:

$$D = \frac{4.375}{12} = 0.364 \text{ ft.}$$

$$h = 0.23\left(\frac{20}{0.364}\right)^{0.25} = 0.63 \text{ Btu per hour per square foot per degree Fahrenheit.} \quad (7)$$

This result may not be deemed conservative inasmuch as the expression is for *still* air. If, however, the air is not still, but flows at approximately 5 mph or 7 ft per second the mass velocity corresponds to:

$$G = 7 \times 0.07 \times 3600 = 1770 \text{ lb air per hour per square foot.}$$

A magnitude of $k = 0.014$ Btu per hour per square foot per degree Fahrenheit for one foot thickness applied to Case 4 yields:

$$h = 0.45\left(\frac{0.014}{0.364}\right) + 0.178\,(1770)^{0.56}\left(\frac{0.014}{0.364}\right)^{0.44}$$

$$= 0.017 + 2.8 = 2.8 \text{ Btu per hour per square foot per degree Fahrenheit.}$$

This conductance is based on 1 sq ft of outside lagging area. Thus, since there are $\pi \times \frac{4.375}{12} = 1.14$ sq ft of outside lagging area associated with 1 ft length of pipe:

$$R_c = \frac{1}{2.8 \times 1.14} = 0.312 \text{ hr degree Fahrenheit per Btu.}$$

The radiation resistance, R_r, which acts in parallel with the convection resistance, R_c, for the transfer of heat to the surface of the insulation, may