rate of discharge, as well as the volume, increases from zero to a maximum, as shown by the solid section of the curve in Fig. 6; thereafter, as p_2/p_1 is decreased further, the discharge is constant, as indicated by the horizontal line. The value of p_2 at the maximum point is called the critical pressure, or p_c, and from Equation 27 it is seen that p_c is approximately 53 per cent of p_1 when air is flowing.

To find the velocity at the critical pressure, it is assumed that the upstream velocity V_1 in Equation 22 is so small as to be negligible. Using the subscript c to indicate conditions at the critical point,

$$\frac{V_c^2}{2g} = \left(\frac{k}{k-1}\right)(p_1v_1 - p_cv_c) \quad or,$$

$$V_c = \sqrt{\left(\frac{2gk}{k-1}\right)(p_1v_1 - p_cv_c)} \tag{37}$$

Substituting Equations 23 and 36, and rearranging, Equation 37 becomes

$$V_c = \sqrt{\left(\frac{2gk}{k+1}\right)p_1v_1} \tag{38}$$

and

$$V_c = \sqrt{\frac{gkp_c}{\rho_c}} \tag{39}$$

Comparing Equation 39 with Equation 25, it will be seen that the velocity at the throat is equal to the velocity of sound at the critical pressure.

Critical flow is attained only in converging tubes, in nozzles, and in orifices with a well-rounded approach. It does not occur in sharp-edged orifices or in nozzles having an expanding outlet section. The so-called critical flow prover uses this property of constant rate of flow above the critical pressure, and finds application as a flow regulator and a quantity-rate meter; in either case, the theoretical rate of flow may be computed from Equation 38, multiplying V_c by the area of the constriction to obtain the volume rate of flow.

In developing the working equations for orifices and nozzles, it is customary to start with

$$V_2^2 - V_1^2 = 2gh_f \tag{40}$$

This may be derived from the Bernoulli equation or from the relations of falling bodies. Now, since

$$A_1V_1 = A_2V_2 = Q_s \tag{41}$$

in which Q_s is the discharge rate in cubic feet per second,

$$\frac{Q_s^2}{A_2^2} - \frac{Q_s^2}{A_1^2} = 2gh_f \tag{42}$$

Transposing,

$$Q_s = \frac{A_1A_2}{\sqrt{A_1^2 - A_2^2}}\sqrt{2gh_f} \tag{43}$$

or

$$Q_s = A_2\frac{1}{\sqrt{1-\left(\frac{A_2}{A_1}\right)^2}}\sqrt{2gh_f} \tag{44}$$

The central term on the right hand side of Equation 44 will be recognized as the *velocity of approach factor*, so that

$$Q_s = A_2\frac{1}{\sqrt{1-\beta^4}}\sqrt{2gh_f} \tag{45}$$

Actual Flow Through Orifices and Nozzles

The actual rate of flow through an orifice, nozzle, or Venturi tube is rarely equal to the theoretical, and generally the actual rate is less than the theoretical. In the case of the nozzle and Venturi tube, this is due to losses from wall roughness, fluid friction, and turbulence during the expansion in the section following the throat. While wall roughness is not a factor in a sharp-edged orifice, fluid friction and turbulence are important, as is the fact that the discharge contracts to a degree variable with the ratio of outlet to inlet pressure after leaving the orifice, so that the limiting area is somewhat less than the opening in the orifice plate. Accordingly, Equation 45 must be modified by a correction factor, C. Usually, the velocity of approach factor is included with this correction factor, and, if

$$K = C\frac{1}{\sqrt{1-\beta^4}}, \tag{46}$$

$$Q_s = KA_2\sqrt{2gh_f}\,. \tag{47}$$

Multiplying by 3600 to convert from cubic feet per second to cubic feet per hour and converting area in square feet to diameter in inches, gives

$$Q_f = 3600\,K\frac{\pi D_2^2}{4\times 144}\sqrt{2gh_f}$$

or

$$Q_f = 19.635\,KD_2^2\sqrt{2gh_f} \tag{48}$$

where

Q_f = rate of flow in cubic feet per hour.
D_2 = the diameter of the orifice or nozzle throat in inches.
K = flow coefficient including correction for velocity of approach.

Equation 48 is a general equation, expressing the flow of any fluid through an orifice or nozzle. Further use of it will be made as other types of flow are discussed.

The differential loss, h_f, is in terms of feet of the fluid flowing through the orifice or nozzle. In the case of a gas flowing, where it is customary to read the differential pressure in inches of water, feet of gas must be converted to inches of water. Since dry air at 32 F and 14.7 psi absolute pressure weighs 0.0807 lb per cubic foot, the weight of a cubic foot of any other kind of gas under the same conditions is 0.0807 G, where G is the specific gravity of the gas referred to air. Water weighs 62.37 lb per cubic foot at 60 F. Using also the relation of 12 in. in 1 ft,

$$h_f = \frac{h_w}{12}\times\frac{62.37}{0.0807G} \tag{49}$$

in which h_w is the differential pressure in inches of water.