Following the electrical analogy, when there is a thermal current flowing through several *resistances in series*, the resistances are additive:

$$R_T = R_1 + R_2 + R_3 + \cdots + R_n \tag{7}$$

Similarly, conductance is the reciprocal of resistance, and for heat flow through several *resistances in parallel*, the conductances are additive:

$$C_T = \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n} \tag{8}$$

Practical Heat Transfer Problems

The use of these relations for resistance and conductance makes possible the solution of many practical heat transfer problems. As discussed in Chapters 9, 28 and 36, the practical analyses of heat transfer in building walls, in fin-tube coils and in pipe coverings, are usually computed by this method. The same resistance analysis may be applied to complicated

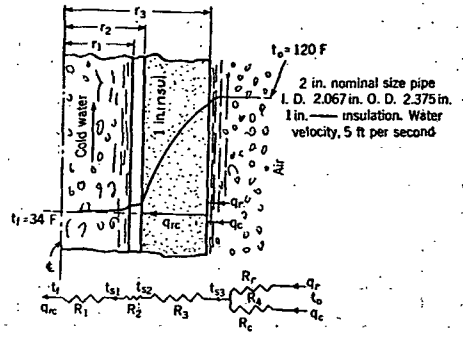


FIG. 8. HEAT TRANSFER CONDITIONS IN AN INSULATED COLD WATER LINE

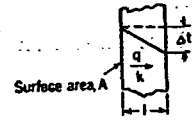
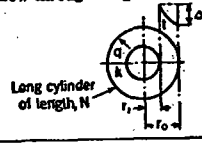
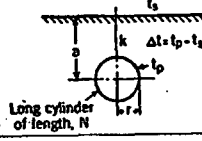
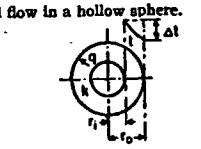
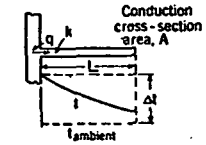
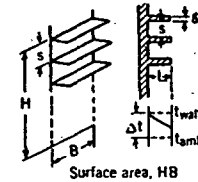
steady-state *conduction* problems. Table 6 gives the resistances in six common cases of steady-state conduction.

A complete analysis by the resistance method is well illustrated by considering the heat transfer from the air outside to the cold water inside of an insulated pipe. The temperature gradients and the nature of the resistance analysis are indicated by the two sketches of Fig. 8.

Since air is sensibly transparent to radiation, there will be some heat transfer by both radiation and convection to the outer insulation surface. The mechanisms act in parallel on the air side. The total transfer by radiation and convection then passes through the insulating layer and the pipe wall by thermal conduction, and thence by convection and radiation into main cold water streams. (Radiation is not significant on the water side as liquids are sensibly opaque to radiation, although water transmits energy in the visible region). The contact resistance between the insulation and the pipe wall is presumed to be equal to zero.

Referring to Fig. 8, the heat transferred for a given length N of pipe, q_{rc}, Btu per hour, may be thought of as flowing through the parallel resistances R_r and R_c, associated with the insulation surface radiation and convection transfer. Then the flow is through the resistance offered tc thermal conduction by the insulation, R_3, through the pipe wall resistance

TABLE 6. SOLUTIONS FOR SOME STEADY-STATE THERMAL CONDUCTION PROBLEMS[a, b]

No.	System	Expressions for the resistance R entering into the equation: $q = \Delta t/R$ (Btu per hour)
1.	Flat wall or curved wall if curvature is small (wall thickness less than 0.1 of inside diameter). 	$R = \frac{L}{kA}$
2.	Radial flow through a right circular cylinder. 	$R = \frac{\log_e \frac{r_o}{r_i}}{2\pi kN}$ (See footnote c).
3.	The buried cylinder. 	$R = \frac{\log_e\left(\frac{a+\sqrt{a^2+r^2}}{r}\right)}{2\pi kN} = \frac{\cosh^{-1}\left(\frac{a}{r}\sqrt{1+\frac{r^2}{a^2}}\right)}{2\pi kN}$ For $\frac{a}{r} \geq 3$, a satisfactory approximation is: $R = \frac{\log_e \frac{2a}{r}}{2\pi kN} = \frac{\cosh^{-1}\frac{a}{r}}{2\pi kN}$
4.	Radial flow in a hollow sphere. 	$R = \frac{\frac{1}{r_i} - \frac{1}{r_o}}{4\pi k}$
5.	The straight fin or rod heated at one end. 	$R = \frac{m}{h_o p \tanh mL}$ (see footnotes d and e). For $ml > 2.3$, $\tanh mL \approx 1$ $m = \sqrt{h_o p/kA}$ A = conduction cross-section area. p = perimeter of cross-section A. h_o = unit conductance to the surroundings from the fin surface. k = thermal conductivity fin material. Δt = wall temperature–ambient temperature
6.	Finned surface of area HB. 	$R = \frac{(s+\delta)}{h_o\left(\frac{2}{m}\tanh ml + s\right)HB}$ $m = \sqrt{\frac{h_o p}{kA}} = \sqrt{\frac{2\,h_o}{k\delta}}$ Δt defined as in Case 5 above.

[a] The dimensions to be employed in these solutions are: length of dimension p, L, r = feet; units of k = Btu per (hour) (square foot) (Fahrenheit degree for *one foot thickness*); units of h, Btu per (hour) (square foot) (Fahrenheit degree); units of area, A = square feet.

[b] The thermal conductivity, k, in these solutions should be taken at the average material temperature.

[c] $\log_e x = 2.303 \log_{10} x$.

[d] This expression can also be employed as an approximation for tapered fins or of annular fins by employing average magnitudes of A and p.

[e] tanh is the hyperbolic tangent.