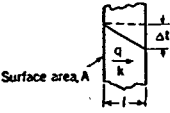
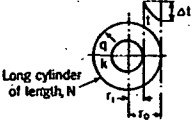
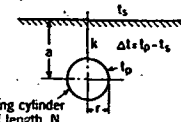
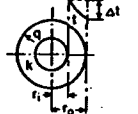
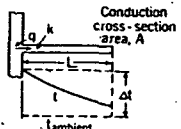
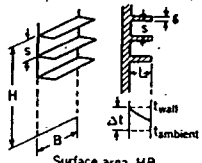


TABLE 5. SOLUTIONS FOR SOME STEADY-STATE THERMAL CONDUCTION PROBLEMS^{a, b}

No.	SYSTEM	Expressions for the resistance R entering into the equation: $q = \Delta t/R$ (Btu per hour)
1.	Flat wall or curved wall if curvature is small (wall thickness less than 0.1 of inside diameter). 	$R = \frac{L}{kA}$
2.	Radial flow through a right circular cylinder. 	$R = \frac{\log_e \frac{r_o}{r_i}}{2\pi k N}$ (See footnote c).
3.	The buried cylinder. 	$R = \frac{\log_e \left(\frac{\Delta t + \sqrt{\Delta t^2 + r^2}}{r} \right) \cosh^{-1} \left(\frac{\Delta t + \sqrt{\Delta t^2 + r^2}}{r} \right)}{2\pi k N}$ For $\frac{\Delta t}{r} \geq 3$, a satisfactory approximation is: $R = \frac{\log_e \frac{2\Delta t}{r}}{2\pi k N} = \frac{\cosh^{-1} \frac{\Delta t}{r}}{2\pi k N}$
4.	Radial flow in a hollow sphere. 	$R = \frac{\frac{1}{r_i} - \frac{1}{r_o}}{4\pi k}$
5.	The straight fin or rod heated at one end. 	$R = \frac{m}{h_o p \tanh mL}$ (see footnotes d and e). For $mL > 2.3$, $\tanh mL \approx 1$ $m = \sqrt{h_o p / kA}$ A = conduction cross-section area. p = perimeter of cross-section A . h_o = unit conductance to the surroundings from the fin surface. k = thermal conductivity fin material. Δt = wall temperature—ambient temperature
6.	Finned surface of area HB . 	$R = \frac{(s + \delta)}{h_o \left(\frac{2}{m} \tanh mL + s \right) HB}$ $m = \sqrt{\frac{h_o p}{kA}} = \sqrt{\frac{2 h_o}{k \delta}}$ Δt defined as in Case 5 above.

^a The dimensions to be employed in these solutions are: length of dimension p, L, r = feet; units of k = Btu per (hour) (square foot) (Fahrenheit degree for one foot thickness); units of h , Btu per (hour) (square foot) (Fahrenheit degree); units of area, A = square feet.

^b The thermal conductivity, k , in these solutions should be taken at the average material temperature (see Table 2).

^c $\log_e z = 2.303 \log_{10} z$.

^d This expression can also be employed as an approximation for tapered fins or of annular fins by employing average magnitudes of A and p .

^e $\tanh$ is the hyperbolic tangent.

The resultant resistance of R_o and R_r acting in parallel (see Fig. 4) can now be evaluated as:

$$\frac{1}{R_t} = \frac{1}{R_o} + \frac{1}{R_r} = \frac{1}{0.312} + \frac{1}{0.75} = 4.54 \text{ Btu per (hour) (Fahrenheit degree).}$$

$$R_t = 0.22 \text{ hr Fahrenheit degree per Btu.}$$

The over-all resistance, R_T , surroundings to cold water, is the sum of $R_1 + R_2 + R_3 + R_4 = 4.1$ hr F deg per Btu for 1 ft length of pipe. Note that the controlling resistances are R_3 and R_4 and that neglect of both R_1 and R_2 would not significantly influence the total resistance, R_T .

On the basis of this resistance calculation the heat transfer from the surroundings to the cold water may be evaluated as:

$$\frac{q_{rc}}{N} = \frac{t_o - t_t}{R_T} = \frac{120 - 34}{4.1} = 21 \text{ Btu per (hour) (foot)}$$

or about 0.175 tons of refrigeration per 100 ft of pipe.

Since the calculation is based on a 1 ft pipe length:

$$q_{rc} = 21 \text{ Btu per hour.}$$

The temperature drops through the various resistances are now readily evaluated by Equation 12 as:

$$t_o - t_{s1} \text{ air to insulation surface} = R_1 q_{rc} = 0.22 \times 21 = 4.6 \text{ F.}$$

$$t_{s1} - t_{s2} \text{ through the insulation} = R_2 q_{rc} = 3.9 \times 21 = 82 \text{ F.}$$

$$t_{s2} - t_{s3} \text{ through the pipe wall} = R_3 q_{rc} = 8.5 \times 10^{-4} \times 21 = 0.02 \text{ F.}$$

$$t_{s3} - t_t \text{ pipe wall to cold water} = R_4 q_{rc} = 2.8 \times 10^{-3} \times 21 = 0.06 \text{ F.}$$

The solution was obtained on the assumption that the air temperature and the outside temperature differed by 20 deg. In order to obtain a slightly better estimate of the rate of heat transfer the numerical solution should be repeated using the temperatures calculated from the previous listed temperature differences.

The foregoing problem serves to illustrate a general method of solving steady-state heat transfer problems. There are many problems which cannot be approximated by steady-state solutions. For instance, the problem of pipe line insulation in transient service; the behavior of automatically controlled thermoflow circuits; or the periodic absorption of solar energy by roof and wall structures during the day and nocturnal radiation to the cold sky at night. The transient heat transfer problem differs from the steady-state in that energy storage rates need to be considered. Thus thermal capacity in addition to resistance effects is significant. The vector sum of the thermal capacitance and resistance is the thermal impedance. It is not within the scope of this chapter to deal with these problems. There are, however, solutions available in graphical form for certain special cases. Also a general approximate method may be employed which is analogous to the treatment of capacity-resistance lumped parameter electrical circuits.

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