Table 1 . . . . Dimensionless Numbers Commonly Used in Heat Transfer[a]

Name	Symbol	Value	Application
Nusselt Number	N_{Nu}	$\left(\frac{hd}{k}\right)$	Natural or Forced Convection, Boiling, Condensation
Reynolds Number	N_{Re}	$\left(\frac{Gd}{\mu}\right)$ or $\left(\frac{\rho VL}{\mu}\right)$	Forced Convection
Prandtl Number	N_{Pr}	$\left(\frac{\mu c_p}{k}\right)$	Natural or Forced Convection, Boiling or Condensing
Stanton Number	N_{St}	$\left(\frac{h}{Gc_p}\right)$	Forced Convection
Grashof Number	N_{Gr}	$\left(\frac{L^3\rho^2\beta g(\Delta t)}{\mu^2}\right)$	Natural Convection
		or $\left(\frac{L^3\rho^2 g(\Delta t)}{T\mu^2}\right)$	(for ideal gases)
Fourier Number	N_{Fo}	$\left(\frac{\alpha\tau}{L^2}\right)$	Unsteady State Conduction
Peclet Number	N_{Pe}	$\left(\frac{Gdc_p}{k}\right)$ or $\left(N_{Re}\right)\left(N_{Pr}\right)$	Forced Convection (small N_{Pr})
Graetz Number	N_{Gz}	$\frac{wc_p}{kL}$	Laminar Convection

[a] A complete list of nomenclature appears at the end of this chapter.

k = thermal conductivity, Btu per (hour) (square foot) (Fahrenheit degree per foot).
A = cross-sectional area normal to flow, square feet.
$\frac{dt}{dx}$ = temperature gradient, Fahrenheit degrees per foot.

Equation 1 states that the heat flow rate q in the x direction is directly proportional to the temperature gradient dt/dx and the cross-sectional area A normal to the flow. The proportionality factor is the thermal conductivity k. The minus sign indicates that the heat flow is positive in the direction of decreasing temperature. Conductivity values are sometimes given in other units, such as Btu per (hr) (sq ft) (F deg per in.), particularly in the case of building materials (Chapter 24), and care must be taken to determine that units are consistent when using Equation 1. Table 2 lists values of the thermal conductivity for a few common substances.

Equation 1 may be integrated along a path of constant heat flow rate to obtain the following simple relation:

$$q = k\frac{A_m}{L_m}\Delta t = \Delta t/R \qquad (2)$$

where

A_m = mean cross-sectional area normal to flow, square feet.
L_m = mean length of heat flow path, feet.
Δt = overall temperature difference, Fahrenheit degrees.
R = thermal resistance, (Fahrenheit degrees) (hour) per Btu.

The *thermal resistance* R is directly proportional to the mean length of the heat flow path L_m, and inversely proportional to the conductivity, k, and the mean cross sectional area normal to the flow, A_m. Equations for the thermal resistances of a few common shapes are given in Table 3. Mathematical solutions to many other heat conduction problems are given by Carslaw and Jaeger.[4] More complicated problems may be solved by graphical or numerical methods.[5,7]

Equation 2 is analogous to Ohm's law for electrical circuits; namely, the thermal current (heat flow) in a thermal circuit is directly proportional to the thermal potential (temperature difference) and inversely proportional to the thermal resistance. This electrical-thermal analogy may be used to obtain experimental results for heat conduction in complex shapes, which resist solution by exact analytical means. This is done by constructing a model from an electrically conducting substance (both liquids and solids have been used), impressing voltages along its boundaries corresponding to the temperatures, and measuring the flow of electric current. This technique (and others which are based on other analogies) are discussed in detail by Schneider.[7] The thermal circuit concept is also useful for attacking problems involving transient heat flow or combined conduction, convection and radiation, as discussed in later sections of this chapter.

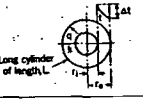
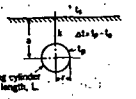
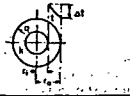
Table 2 . . . . Approximate Thermal Conductivities[a] (See Also Chapters 19, 24, and 69)

Conductivity k = Btu per (hour) (square foot) (Fahrenheit degree per foot)

Substance	k	Substance	k
Refrigerant 12 (Vapor)	0.0057	Snow	0.1–0.3
Air	0.015	Glass	0.2–0.7
Hydrogen	0.10	Soil	0.6–1.0
Refrigerant 12 (Liquid)	0.045	Steel	26
Water	0.35	Brass (70–30)	58
Mercury	4.8	Aluminum	118
Corkboard	0.025	Copper	220

[a] Thermal conductivities depend to some extent on temperature. The above magnitudes are approximate only.

Table 3 . . . . Solutions for Some Steady-State Thermal Conduction Problems[a,b]

No.	System	Expressions for the Resistance R Entering into the Equation: $q = \Delta t/R$ (Btu per hour)
1.	Flat wall or curved wall if curvature is small (wall thickness less than 0.1 of inside diameter). 	$R = \frac{L}{kA}$
2.	Radial flow through a right circular cylinder. 	$R = \frac{\log_e \frac{r_o}{r_i}}{2\pi kL}$
3.	The buried cylinder. 	$R = \frac{\log_e\left(\frac{a+\sqrt{a^2+r^2}}{r}\right)}{2\pi kL} = \frac{\cosh^{-1}\left(\frac{a}{r}\sqrt{1+\frac{r^2}{a^2}}\right)}{2\pi kL}$ For $\frac{a}{r} \geq 3$, satisfactory approximation is: $R \simeq \frac{\log_e \frac{2a}{r}}{2\pi kL} = \frac{\cosh^{-1}\frac{a}{r}}{2\pi kL}$
4.	Radial flow in a hollow sphere. 	$R = \frac{\frac{1}{r_i} - \frac{1}{r_o}}{4\pi k}$

[a] The units to be employed in these solutions are: length L, r, a = feet; conductivity k = Btu per (hour) (square foot) (Fahrenheit degree for *one foot thickness*); heat transfer coefficient h, Btu per (hour) (square foot) (Fahrenheit degree); area, A = square feet.

[b] The thermal conductivity, k, in these solutions should be taken at the average material temperature.

THERMAL RADIATION

Radiation is one of the basic mechanisms by which energy is transferred between regions of different temperature. It is distinguished from conduction and convection by the fact that it does not depend upon the presence of an intermediate material to act as a carrier of energy. On the contrary, a radiation-transfer process between two regions is impeded by the presence of a material in the space between. The radiation energy transfer process is explained as the consequence of energy-carrying electromagnetic waves. These waves are emitted by atoms and molecules as the result of various changes in their energy content. The amount and characteristics of the radiant energy emitted by a quantity of material depend upon the nature of the material, its microscopic arrangement, and its absolute temperature. Although the rate of emission of energy is independent of the surroundings, the *net* energy transfer rate depends upon the temperatures and spatial relationships of the various materials involved.

Black Body Radiation

The rate of thermal radiant energy emission by a surface is dependent upon its absolute temperature. The relation between the energy-emission rate and the temperature is very simple if the surface is *black*: A surface is called black if it can absorb all incident radiation. The total rate of energy emission per unit area of black surface, W_b, to the hemispherical region above it is given by the *Stefan-Boltzmann* law.

$$W_b = \sigma T^4 \qquad (3)$$

The constant, σ, in English units is 0.1713×10^{-8} Btu per (hr) (sq ft) (Rankine deg to the fourth power).

This total amount of radiation is distributed over a spectrum of wave lengths. The generally accepted wave length distribution law for a black surface at temperature T is the one developed by *Planck*:

$$W_{b\lambda} = \frac{C_1\lambda^{-5}}{e^{C_2/\lambda T} - 1} \qquad (4)$$