also be assumed that the flow pattern is essentially constant along the tube so that slug flow does not occur. These are the assumptions usually made in calculating pressure drops for boilers and evaporators. The case of refrigerants flowing in capillary tubes is discussed in Chapter 45 of this volume.

The major problem in two-phase flow is that of predicting the flow pattern which will occur when various sets of variables are specified. Although this problem is at present unsolved, it is known that the following kinematic flow patterns exist for flow in horizontal conduits:[4]

1. Stratified flow—Liquid in the lower part of the conduit and gas in the upper part with a smooth interface between the two phases.
2. Wave flow—Similar to stratified flow except for wavy interface due to higher relative gas velocity.
3. Wave-mist flow—Wave flow in which high gas velocity causes atomization to take place at the interface.
4. Plug flow—Vapor moves in bubbles or plugs along the upper side of the pipe.
5. Slug flow—Slugs or splashes of liquid move at higher velocity than the bulk of the liquid. Pressure fluctuations characterize this type of flow.
6. Mist-annular flow—Relative gas velocity is high enough that liquid is partially atomized in the gas phase and partly flowing in the annular film along the pipe wall.
7. Froth flow—Gas is dispersed in fine bubbles throughout the liquid.

In addition to their classification kinematically, two-phase flow patterns can be classified dissipatively. Since each phase can individually be in a state of either laminar or turbulent flow, there are four dissipative classifications. These are

1. Turbulent liquid-turbulent vapor (*t-t*).
2. Viscous liquid-turbulent vapor (*v-t*).
3. Turbulent liquid-viscous vapor (*t-v*).
4. Viscous liquid-viscous vapor (*v-v*).

In the example problem to be discussed later, it will be shown how the dissipative flow pattern can be established.

There are several methods available for calculating pressure drop in two-phase flow. A review of these methods shows that the early semi-empirical work of Lockhart and Martinelli[5] and the extension by Martinelli and Nelson[6] gives values which are as reliable as those determined with more recent analytical methods.[7,8,9] For example, Hatch and Jacobs[10] conclude that the Martinelli-Nelson method can be used to predict, approximately, the two-phase pressure drop of Refrigerant 11. For Refrigerant 22 at pressures around 100 psia, Altman, Norris, and Staub[11] have shown that the Martinelli-Nelson method correlated 80 percent of their pressure drop data within +5 and −20 percent.

It is very important to realize that the value of the pressure drop calculated for a two-phase flow is quite dependent on the computational method and assumptions employed. If the pressure drop across the tube is appreciable (say, as much as 10 percent of the inlet pressure) then a method of successive approximations should be employed in which the pressure and perhaps position along the tube are changed from one approximation to the next. Thus, even if one basis (say, the Lockhart-Martinelli basis) is adopted for making the two-phase flow calculation, it is still possible to get different answers for the pressure drop, depending on the computational approximations and assumptions made. An illustration of the order of magnitude of the effects discussed above is provided in the example at the end of this section.

The foregoing discussion indicates that, at the present time, the choice of a pressure drop model and computational method is rather arbitrary with only an approximate value for the pressure drop being obtainable. Thus, the following procedure should be regarded as an interim procedure to be followed only until better data and methods are formulated.

In two-phase pressure drop computations for horizontal tubes, it is conventional to construe the two-phase pressure drop as consisting of the sum of a frictional component and an acceleration component.

$$\Delta p_{TP} = \Delta p_{fr} + \Delta p_a \qquad (27)$$

Martinelli and Nelson have presented two different methods for evaluating Δp_a in terms of properties at the inlet and exit of the tube. If it is assumed that a slip exists between the phase velocities (separated flow), then Δp_a is found to be a function of void fraction, quality, and fluid density. Thus if the existence of slip between the phases is assumed, then void fraction data must be available.

If, on the other hand, it is assumed that both phases have the same velocity (homogeneous flow), a simplified expression is obtained for Δp_a which does not depend on void fraction. Assuming homogeneous flow, the acceleration component Δp_a is given by:

$$\Delta p_a = \frac{16w^2}{\pi^2 D^4}\left[\frac{(1-x_e)}{\rho_{fe}} - \frac{(1-x_i)}{\rho_{fi}} + \frac{x_e}{\rho_{ge}} - \frac{x_i}{\rho_{gi}}\right] \qquad (28)$$

where

Δp_a = acceleration pressure drop, pounds per square foot.
w = two phase mass flow rate, slugs per second.
ρ_f = density of saturated liquid, slugs per cubic foot.
ρ_g = density of saturated vapor, slugs per cubic foot.
x = vapor quality.

The subscripts e and i refer to the tube exit and inlet, respectively.

In order to calculate the frictional component Δp_{fr}, it is in general necessary to know the distribution of quality, x, along the tube length, L. This requires a knowledge of the heat transfer coefficient which, in turn, depends on the pressure drop. Thus, the general problem is one in which the heat transfer is coupled to the hydraulics.

Martinelli and Nelson have presented an approximate method for calculating pressure drop which removes the coupling between heat transfer and pressure drop. This was done simply by assuming that the quality, x, varies linearly along the tube length, L. With this assumption, the frictional component, Δp_{fr}, is given by

$$\Delta p_{fr} = \Delta p_o\left(\frac{1}{x_e - x_i}\int_{x_i}^{x_e} \phi_o^2\, dx\right) \qquad (29)$$

where

Δp_o = frictional pressure drop for all liquid flow, pounds per square foot.
ϕ_o^2 = Martinelli-Nelson frictional multiplier.

The parameter ϕ_o^2 is the Martinelli-Nelson multiplier function depending on pressure and quality. That is:

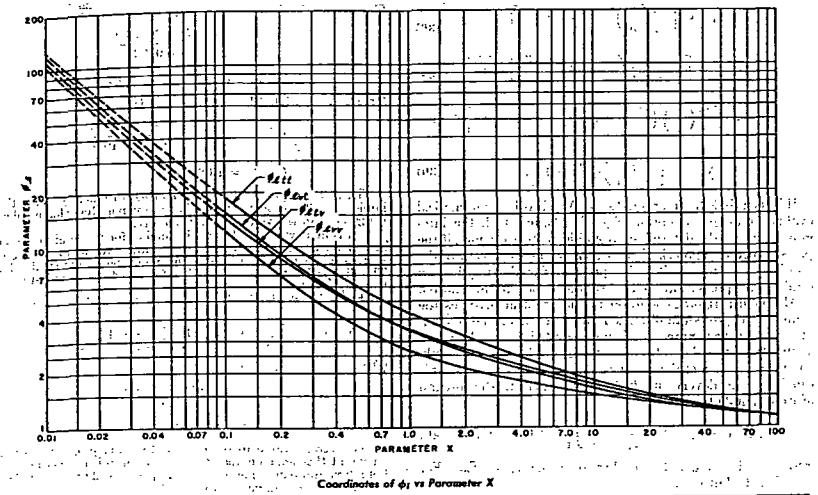
$$\phi_o^2 = f(p, x) \qquad (30)$$

The quantity Δp_o is simply the single phase frictional pressure drop for completely liquid flow at a mass flow rate equal to the two-phase mass flow rate, w. This is given by Equation 22, which becomes:

$$\Delta p_o = \frac{f_o L}{D}\frac{\rho_f V_o^2}{2} \qquad (31)$$

where

$$V_o = \frac{w}{\rho_f A} \qquad (32)$$



Coordinates of ϕ_t vs Parameter X

X	Turbulent-Turbulent ϕ_t	Turbulent-Viscous ϕ_t	Viscous-Turbulent ϕ_t	Viscous-Viscous ϕ_t	X	Turbulent-Turbulent ϕ_t	Turbulent-Viscous ϕ_t	Viscous-Turbulent ϕ_t	Viscous-Viscous ϕ_t
0.01	128	112	120	105	2.0	3.10	2.62	2.62	2.06
0.02	68.4	58.0	64.0	53.5	4.0	2.38	2.15	2.05	1.76
0.04	38.5	31.0	34.0	28.0	7.0	1.96	1.83	1.73	1.60
0.07	24.4	19.3	20.7	17.0	10	1.75	1.66	1.59	1.50
0.10	18.5	14.5	15.2	12.4	20	1.48	1.44	1.40	1.36
0.2	11.2	8.70	8.90	7.00	40	1.29	1.25	1.25	1.25
0.4	7.05	5.50	5.62	4.25	70	1.17	1.17	1.17	1.17
0.7	5.04	4.07	4.07	3.08	100	1.11	1.11	1.11	1.11
1.0	4.20	3.48	3.48	2.61					

Adapted from Reference 5.

Fig. 6 .... Relationship Between Parameter ϕ_t and Parameter X

The friction factor f_o is determined in the conventional way (say from Fig. 5) from a chart of f_o versus Reynolds number $(N_{Re})_o$

where

$$(N_{Re})_o = \frac{V_o D}{\nu_f} \qquad (33)$$

Martinelli and Nelson have presented curves for ϕ_o^2 as a function of pressure for water. Although similar curves could be prepared for a specific refrigerant, this would involve a great deal of calculational work. An alternate procedure is to use the Lockhart-Martinelli data which is independent of any particular fluid and is pressure dependent only through the fluid properties used. This will be illustrated in the example problem.

The relation between the Martinelli-Nelson Multiplier ϕ_o^2 and the Lockhart-Martinelli multiplier ϕ_t^2 (presented in Fig. 6) can be shown to be:

$$\phi_o^2 = \phi_t^2(1-x)^{1.8} \qquad (34)$$

Although Martinelli and Nelson used an exponent of 1.75, the value 1.8 should be used if Fig. 6 is used for ϕ_t^2.

Then, the frictional component of pressure drop becomes:

$$\Delta p_{fr} = \Delta p_o\left(\frac{1}{x_e - x_i}\int_{x_i}^{x_e} \phi_t^2(1-x)^{1.8}\, dx\right) \qquad (35)$$

where ϕ_t is a function of the type of (dissipative) flow pattern and a parameter $\underline{X}$. See Fig. 6. The parameter $\underline{X}$ is also a function of the type of flow pattern, the quality x, and the fluid properties (which makes $\underline{X}$ slightly pressure dependent). The parameter $\underline{X}$ is defined below for the four possible dissipative flow patterns: