PERIODIC AND TRANSIENT HEAT FLOW

The foregoing data and examples dealt with steady-state heat transfer (not varying with time). In most practical heat transfer problems the heat flow depends upon time. Such cases can usually be divided into two classes: periodic and transient. Periodic heat transfer repeats periodically in time. Transient heat transfer exhibits no periodicity. Graphical, analytical and numerical methods are available for solving transient or periodic heat flow problems.[4,5,10,11,12] Graphical and numerical methods are the most versatile, and can be applied with minimum mathematical training.

A large number of analytical solutions for the case of heat conduction in variously shaped solids are available in the literature. Table 7 gives a summary of the cases reported and tabulated. Many more analytical solutions are available in the form of infinite series,[11,12,18,19] but are not tabulated. Certain complex cases may be treated by combining the simple analytical solutions as discussed in Reference 17. (See also Reference 20).

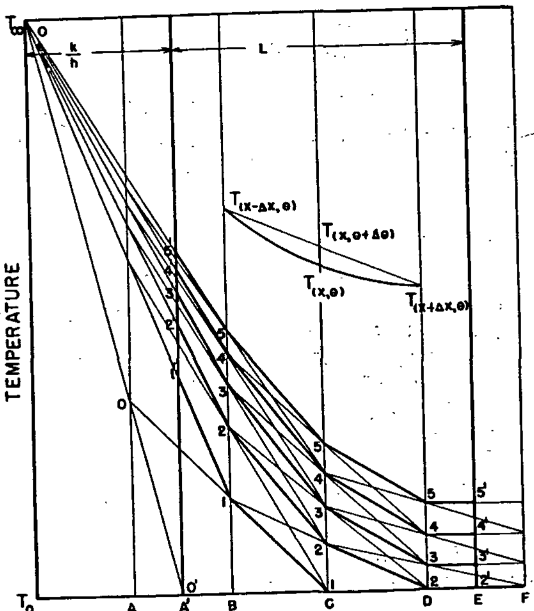
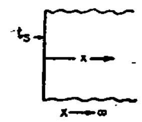
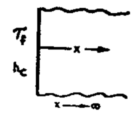
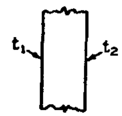
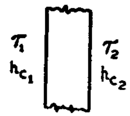


Fig. 9. Example of a Graphical Solution to a Problem in Transient Heat Conduction.

Frequently, transient heat flow problems in one dimension have boundary conditions which make the problem difficult to treat analytically. In such cases, recourse may be made to a graphical method of solution sometimes called the Schmidt method.[4,10,19,21,22] This method will be briefly outlined for the case of transient heat flow in a slab insulated on one face, and suddenly exposed on the other face through a fixed thermal resistance to a higher temperature. The technique is general, however, and methods may be devised for any boundary conditions,[5,21] and also, for one dimensional (radial) heat flow in spheres and cylinders.[22,23]

Table 7. Analytical Solutions for Heat Conduction in Variously Shaped Solids

Shape of Solid	Boundary Conditions	Data Available in Graphs
Semi-infinite 	Surface temperature suddenly changed.	Temperature distribution in solid as a function of time. References: (4) p. 37, (5) p. V-28; (10) p. 254; (13) p. 46.
		Heat flow from surface as a function of time. References: (10) pp. 256, 267; (13) p. 47
	A steady flow of heat is suddenly applied to the surface.	Temperature distribution as a function of time. Reference: (10) p. 257.
	The surface temperature has been varying sinusoidally with time for a long time.	Temperature distribution as a function of time. Reference: (10) p. 296.
		Heat flow from surface as a function of time. Reference: (10) p. 296.
Semi-infinite with fluid at free surface. 	The temperature of the fluid in contact with the surface has a sudden change in temperature. (The surface conductance is constant).	Temperature distribution as a function of time. References: (4) p. 37; (5) pp. V-45, 46, 47.
	The temperature of the fluid in contact with the surface has been varying sinusoidally with time for a long time. (The surface conductance is constant.)	Temperature distribution as a function of time. Reference: (10) p. 298.
		Heat flow from the surface as a function of time. Reference: (10) p. 298.
Slab. 	The temperatures t_1 and t_2 are suddenly changed from the initial uniform slab temperature to a new temperature. (The case where the surface on one side is insulated is treated by taking the case of a slab of twice the given thickness since the midplane has no heat flow due to symmetry.)	Temperature distribution as a function of time. References: (5) p. V-12; (10) p. 265.
	The temperatures t_1 and t_2 suddenly begin to increase as linear functions of time. The slab is initially at uniform temperature. (The case where one surface is insulated against heat flow is treated as noted above.)	Temperature distribution as a function of time. Reference: (10) p. 268.
	The temperature at both surfaces has been varying sinusoidally for a long time.	Temperature distribution as a function of time. Reference: (10) p. 300.
		Heat flow from the surface. Reference: (10) p. 303.
Slab immersed in a fluid with constant conductance between fluid and slab surface. 	The temperature of the fluid is suddenly changed from the initial uniform slab temperature. (If one surface is insulated against heat flow, see above.)	Temperature distribution as a function of time. References: (4) pp. 32, 33, 34, 35; (5) pp. V-9, 10, 35, 42; (10) pp. 274, 284; (13) p. 106.
		Heat flow from the surface as a function of time. References: (5) p. V-10; (10) p. 274; (13) p 107.
	The temperature of the fluid at one surface varies as a periodic function of time while the temperature of the fluid at the other surface is constant. The conductances need not be the same on both sides. (The variations in temperature are expressible as a Fourier series.)	Temperature distribution as a function of time. Reference 14.
		Heat flow at the surface as a function of time. Reference 14.