STEADY FLOW ENERGY EQUATION

It was previously stated that, in steady flow, the energy convected by the fluid at any section is the sum of (*a*) kinetic energy due to velocity; (*b*) gravitational energy due to elevation; (*c*) enthalpy due to the condition of pressure, temperature and composition of the fluid. A more detailed discussion of item (*a*) is in order.

Kinetic Energy

There are reasons to believe that the so-called *velocity pressure* h_v read by a Pitot tube is simply the kinetic energy per unit volume of the fluid immediately upstream from the tube, as application of Bernoulli's Equation suggests. Thus (see Equation 3, Chapter 34).

$$V = 1097.3 \sqrt{\frac{h_v}{d}} \tag{25}$$

where

V = velocity, feet per minute.
h_v = velocity pressure, inches of water at 60 F.
d = density of fluid, pounds per cubic foot.

In the case of flow through a duct, the velocity pressure is found to vary considerably over the section and a traverse has to be made. The cross-sectional area of the duct is divided into a number of equal concentric areas, and measuring stations are located at centroidal points in each area along two perpendicular diameters. Usually the ultimate object is to determine an *average* velocity $\overline{V}$ from which the weight of fluid crossing the section per unit time can be obtained on multiplying by the cross-sectional area of the duct and by the density of the fluid. This is obtained by simply averaging the square roots of all measured velocity pressures as follows:

$$\overline{V} = \frac{1097.3}{\sqrt{d}} \left(h_v^{1/2} \right)_{av} \tag{26}$$

where

$\overline{V}$ = average velocity, feet per minute.
$(h_v^{1/2})_{av}$ = arithmetic average of the square roots of all measured velocity pressures, inches of water at 60 F.

But the item of present importance is the *average kinetic energy* convected with each pound of fluid. Consistently with the previous discussion, this can be shown to be

$$\overline{KE} = 0.006678\, v \frac{(h_v^{3/2})_{av}}{(h_v^{1/2})_{av}} \tag{27}$$

where

$\overline{KE}$ = average kinetic energy, Btu per pound.
v = specific volume, cubic feet per pound.
$(h_v^{3/2})_{av}$ = arithmetic average of the 3/2-powers of all measured velocity pressures, inches of water at 60 F.

If the velocity pressure were uniform over the section, Equations 26 and 27 could be combined to give

$$\overline{KE} = \left(\frac{\overline{V}}{13{,}430} \right)^2 \tag{28}$$

But, it is interesting to note that if the velocity varies *parabolically* from zero at the walls to maximum at the center as it does in the case of purely viscous flow in a circular duct, *then the average kinetic energy is twice that given by* Equation 28.

Example 20. If 2000 cfm of air flows through an 8 in. diameter circular duct, find the average kinetic energy per pound of air.

Solution. The cross-sectional area of the duct is 0.349 sq ft; hence the average flow velocity is 5730 fpm. If the velocity were uniform over the section, the average kinetic energy would be $(5730 \div 13{,}430)^2 = 0.182$ Btu per pound. But it is more likely that the actual distribution of velocity would approximate that characteristic of viscous flow; hence the average kinetic energy would be more nearly $2 \times 0.182 = 0.364$ Btu per pound.

Gravitational Energy

The potential energy due to elevation Z (feet) above any convenient datum is simply $Z \div 778.3$ Btu per pound of fluid. In the case of moist air,

$$\overline{PE} = \frac{\overline{Z}\,(1 + W)}{778.3} \tag{29}$$

where

$\overline{PE}$ = average potential energy, Btu per pound dry air.
$\overline{Z}$ = average elevation, feet.
W = humidity ratio, pound water per pound dry air.

Enthalpy

No further discussion of enthalpy is required. It may be well to emphasize, however, that enthalpies have been figured on the basis of one pound of dry air.

Heat and Shaft Work

Between any two sections 1 and 2 in an apparatus through which steady flow occurs, there may be heat *absorbed* from outside, ${}_1q_2$, Btu per pound of dry air, and shaft work *removed* to outside, ${}_1l_2$, Btu per pound of dry air. If heat is actually rejected to outside, ${}_1q_2$ is intrinsically negative; and if shaft work is actually put in from outside ${}_1l_2$, is intrinsically negative.

Steady-flow Energy Equation

A complete energy accounting takes the form of Equation 30 which is usually referred to as the steady-flow energy equation.

$${}_1q_2 = (h_2 + \overline{KE}_2 + \overline{PE}_2) - (h_1 + \overline{KE}_1 + \overline{PE}_1) + {}_1l_2 \tag{30}$$

where

${}_1q_2$ = heat *added* from outside *between* sections 1 and 2, Btu per pound dry air.
h_2 = enthalpy of the mixture *at* section 2, Btu per pound dry air.
$\overline{KE}_2$ = average kinetic energy *at* section 2, Btu per pound dry air.
$\overline{PE}_2$ = average potential energy *at* section 2, Btu per pound dry air.