For instance, in a square duct, 1 ft on a side, handling air, the hydraulic radius is ¼ or 0.25. If the same duct is handling water, flowing 9 in. deep, the hydraulic radius is 0.75/2.5 or 0.30. Note in this latter case that the wetted perimeter does not include the distance across the free surface.

In the case of a round pipe

$$R_H = \frac{\pi d^2/4}{\pi d} = \frac{d}{4} \text{ or } d = 4R_H \tag{12}$$

Substituting Equation 12 in Equation 6,

$$N_{Re} = \frac{4R_H V\rho}{\mu} \tag{13}$$

and in the flow Equation 5,

$$h_f = \frac{flV^2}{8gR_H} \tag{14}$$

and finally in the compressible fluid flow Equation 10,

$$p_1 - p_2 = p_1\left[1 - \sqrt{1 - \frac{flV_1^2}{4g\,R_H\,p_1v_1}}\right] \tag{15}$$

Equations 13, 14, and 15 may be used to compute the flow in pipes and ducts of non-circular section and in any type of conduit not flowing full. *They should not be used when the flow is laminar.*

FLOW OF COMPRESSIBLE FLUIDS

The energy equation for the flow of compressible fluids, as represented by the gases, is derived from Equation 1. Assuming that no heat is transferred to the fluid, that no work is done, and that there is no difference in elevation, Equation 1 becomes

$$\frac{V_1^2}{2g} + Ju_1 + p_1v_1 = \frac{V_2^2}{2g} + Ju_2 + p_2v_2 \tag{16}$$

or, after rearrangement,

$$\frac{V_2^2 - V_1^2}{2g} = p_1v_1 - p_2v_2 + J(u_1 - u_2) \tag{17}$$

Since internal energy is dependent only on temperature,

$$u_1 - u_2 = c_v(T_1 - T_2) \tag{18}$$

where

c_v = the specific heat of the gas at constant volume.

T_1 and T_2 = the absolute temperatures in Fahrenheit degrees at points 1 and 2, respectively.

Substituting Equation 18 in Equation 17,

$$\frac{V_2^2 - V_1^2}{2g} = p_1v_1 - p_2v_2 + Jc_v(T_1 - T_2) \tag{19}$$

Now

$$c_v = \frac{R}{J(k-1)} \tag{20}$$

where

R = the gas constant in the expression.

$$pv = RT \tag{21}$$

k = the ratio of the specific heat at constant pressure to the specific heat at constant volume.

This ratio, k, is used extensively in fluid dynamics; values of k for various gases are given in Table 2.

TABLE 2. RATIO OF SPECIFIC HEAT AT CONSTANT PRESSURE TO SPECIFIC HEAT AT CONSTANT VOLUME FOR COMPRESSIBLE FLUIDS

COMPRESSIBLE FLUID	RATIO $k = c_p/c_v$
Helium and other monatomic gases	1.66
Air and other diatomic gases	1.40
Ammonia and hydrogen sulfide	1.34
Carbon dioxide, methane, natural gas, superheated steam, moist steam down to a quality of 97 per cent	1.28 to 1.32
Sulfur dioxide, ethylene, acetylene	1.24 to 1.26

Substituting Equations 20 and 21 in Equation 19, the energy equation becomes

$$\frac{V_2^2 - V_1^2}{2g} = \frac{k}{k-1}(p_1v_1 - p_2v_2) \tag{22}$$

While this is a convenient form of equation, it does not include all the necessary specifications. If the steady flow process is frictionless and reversible,

$$\frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^k = \left(\frac{\rho_2}{\rho_1}\right)^k \tag{23}$$

By introducing this relation in Equation 22 it is possible to reduce that equation to:

$$\frac{V_2^2 - V_1^2}{2g} = \frac{k}{k-1}\,\frac{p_1}{\rho_1}\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}\right] \tag{24}$$

This form of the equation is applicable not only to flow in pipes, but also to flow through orifices and nozzles.

A significant factor in the flow of compressible fluids is the velocity of sound, V_{so}, which for present purposes will be considered as the velocity at which sound will travel in the fluid at its density at the first or inlet section of the flow system being considered.

The velocity of sound is expressed as

$$V_{so} = \sqrt{\frac{gkp_1}{\rho_1}} \tag{25}$$

This formula may be developed rationally and agrees perfectly with experimental results. Substituting Equation 25 in Equation 24:

$$\frac{V_2^2 - V_1^2}{2} = \frac{V_{so}^2}{k-1}\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}\right] \tag{26}$$