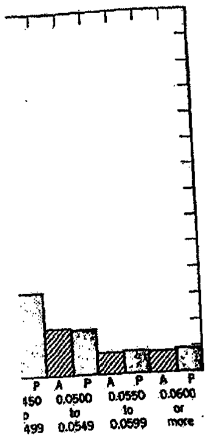
$\left.\begin{matrix}3\\3\end{matrix}\right) = 0.0198.$

:d percentages in selected excellent agreement (see



observations for Johnson S_T

those proposed by Karl be generated as a solution

(6-34)

x^2

ty function $f(x)$ by proper The solution of this equa- lies, including the normal, ype III) distributions di- ı the (β_1, β_2) plane corre- hown in Fig. 6-11. This ates the wide diversity of to select the appropriat

approximation for a given variate, based on knowledge or estimates of β_1 and β_2. The expressions for the probability density functions for the various Pearson distributions are given in References 6-7, 6-9 and 6-10.

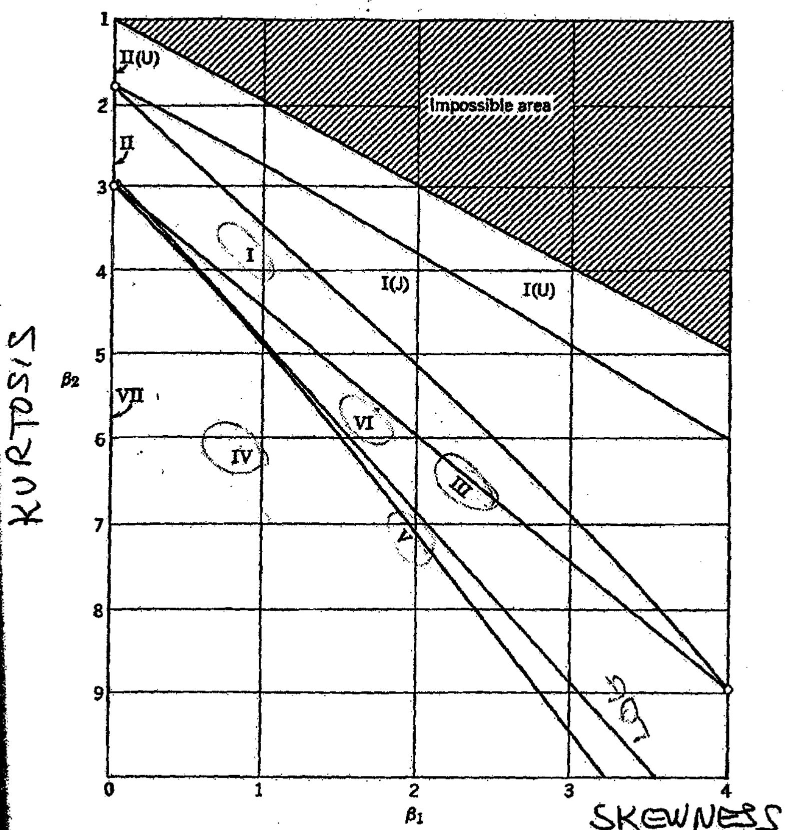


Fig. 6-11 Region in (β_1, β_2) plane for various type Pearson distributions. Letters U and J denote U-shaped and J-shaped distributions. (From E. S. Pearson, Seminars, Princeton University, 1960.)

The descriptions of the procedures for fitting Pearson distributions to data are lengthy, since each family requires solution of a different set of equations. The underlying principles are reviewed in Reference 6-2 and formulae for each family are given in Reference 6-7. These expressions, together with those for estimating the first four moments and β_1 and β_2, may be used to develop a computer program to fit a Pearson distribution

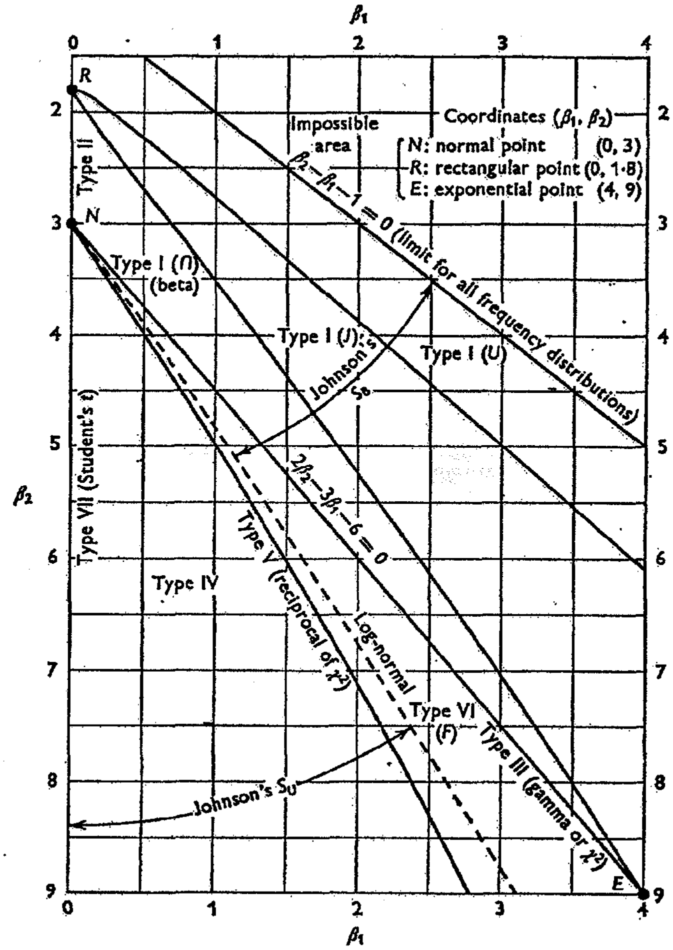


Fig. 7

The following work illustrates the use of the present table in deriving an upper 1 per cent point for a Pearson curve having the correct first four moments. We first extract the following entries from Table 32:

β_2 \ $\sqrt{\beta_1}$	1·5	δ^2	1·6	δ^2	1·7	δ^2	1·8	δ^2
6·4	3·3297		3·4016		3·4748		3·5435	
6·6	3·3239	−4	3·3960	−10	3·4710	−21	3·5452	−44
6·8	3·3177	−3	3·3894	−5	3·4651	−12	3·5425	−28
7·0	3·3112		3·3823		3·4580		3·5370	

Interpolating in each column at $\beta_2 = 6{\cdot}7905$, using the simple Bessel formula (319), we find:

$\sqrt{\beta_1}$	X	δ^2
1·5	3·3180	
1·6	3·3897	40
1·7	3·4654	16
1·8	3·5427	

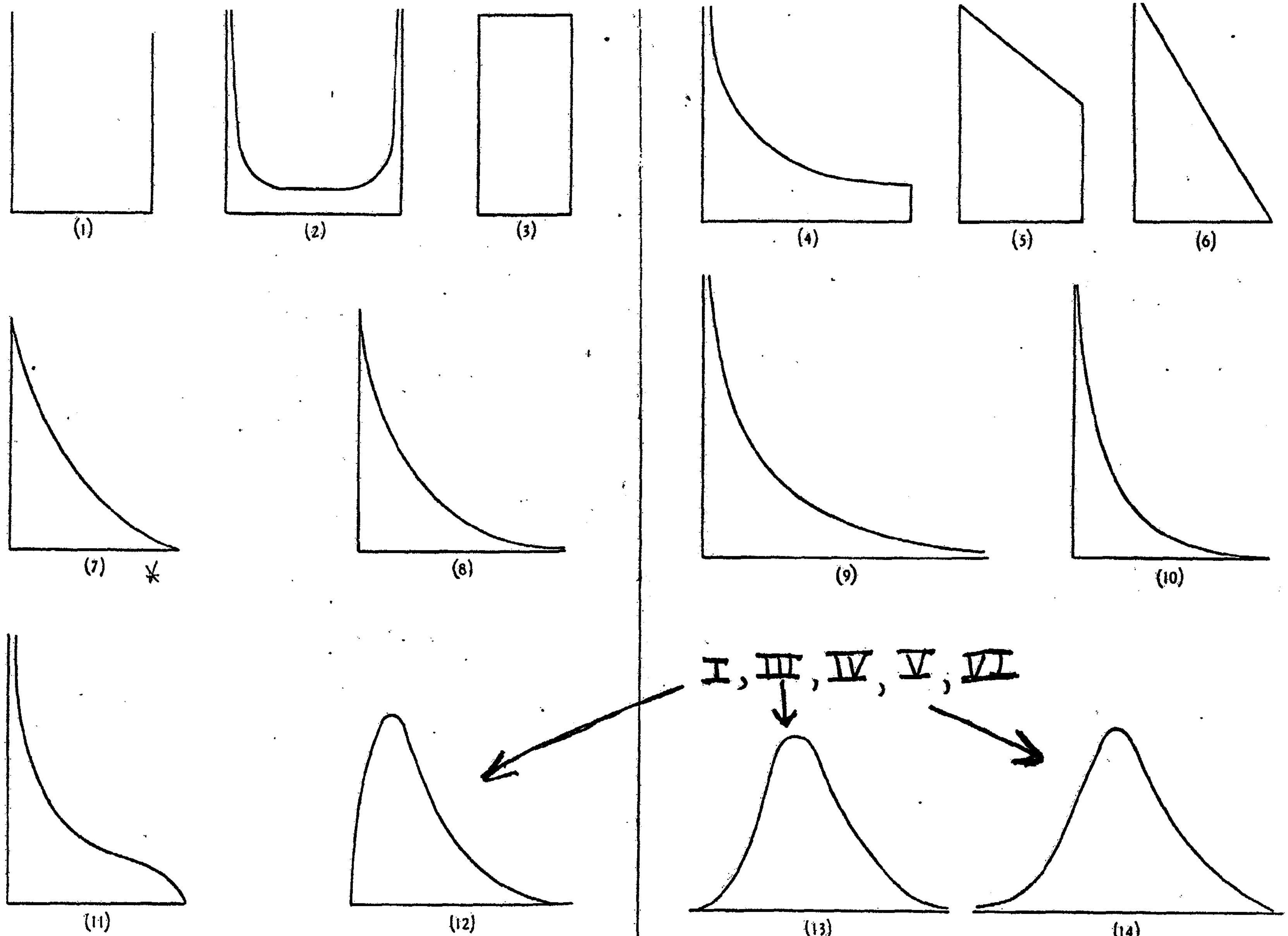


Diagram showing transition from two separated blocks of frequency through U-shaped and J-shaped curves to the more common "cocked hat" shapes.

(1) represents two separated blocks of frequency developing into the U-shaped curve in (2). The horizontal straight line of (3) is, as it were, the bottom piece of the U-curve and the Type VIII curve of (4) is like a part of the U-curve. (5) and (6) are limits when straight lines are reached. (7) is Type IX and (8) is the exponential. The next two curves (9) and (10) are the J-shaped curves of Types III and I, and (11) is Type XII. From this we proceed to Types I, III, IV, V and VI, curves of the "cocked hat" shape, three examples being given in (12), (13) and (14).

(*b*) the standardized values for the terminal or terminals (0 and 100 per cent points), when the curve is of finite range;

(*c*) seven additional percentage points;

(*d*) a much more extensive range of $\sqrt{\beta_1}$, β_2 values.

Tables 31 and 33, based on D. E. Amos's unpublished computations, extend information into the *J*-curve area (see Fig. 7) of the Type I or beta distributions. These two tables will be referred to later in § 19 in comparison with the Johnson S_B curves having $\sqrt{\beta_1}$, β_2 values in this area.

The earlier table took β_1 as argument but it was realized that for interpolation purposes it was better to use $\sqrt{\beta_1}$.

The curves of Karl Pearson's system are all derived as solutions of the differential equation

$$\frac{1}{y}\frac{dy}{dx} = \frac{-(c_1+x)}{c_0+c_1x+c_2x^2}, \tag{168}$$

where the origin is at the mean and the parameters c_0, c_1 and c_2 are functions of $\sqrt{\mu_2} = \sigma$, β_1 and β_2. Using a standard notation (see Elderton & Johnson, 1969) the main curves of the system may be expressed as follows:

Type	Equation	Origin for x	Limits for x
I	$y = y_0 x^{m_1}\left(1-\frac{x}{a}\right)^{m_2}$	At start	$0 \leqslant x \leqslant a$
II	$y = y_0\left(1-\frac{x^2}{a^2}\right)^{m}$	Mean (= mode)	$-a \leqslant x \leqslant a$
III	$y = y_0 x^m e^{-x/a}$	At start	$0 \leqslant x \leqslant \infty$
IV	$y = y_0\left(1+\frac{x^2}{a^2}\right)^{-m} e^{-b\tan^{-1}(x/a)}$	Mean + d*	$-\infty < x < \infty$
V	$y = y_0 x^{-m} e^{-a/x}$	At start	$0 \leqslant x < \infty$
VI	$y = y_0 x^{m_2}\left(1+\frac{x}{a}\right)^{-m_1}$	At start	$0 \leqslant x < \infty$
VII	$y = y_0\left(1+\frac{x^2}{a^2}\right)^{-m}$	Mean (= mode)	$-\infty < x < \infty$

* $d = \frac{1}{2}ab/(m-1)$.

The regions and their boundaries in the β_1, β_2 field with which the different types of solution are associated have been illustrated in Fig. 7. Because all types derive from the solution of (168), their shapes may be described as changing continuously across the boundaries of the regions, although the mathematical forms differ.

17.2 *Illustrations of use of Table* 32

To derive the full benefit from Table 32, it will generally be necessary to use second difference interpolation formulae for both $\sqrt{\beta_1}$ and β_2; the procedure needed lends itself readily to a computer routine. In one example below we illustrate, however, the steps needed in computing with a desk calculator.

Example 48 (taken from Johnson *et al.* 1963, pp. 464–5). Stephens (1963) derived the first four moments of the distribution of G. S. Watson's goodness of fit statistic, U_N^2. For the case of a sample of size $N = 10$, the following results are derived from his expressions (equation (4)):

$$\text{mean} = 0{\cdot}08333, \quad \text{S.D.} = 0{\cdot}05000, \quad \sqrt{\beta_1} = 1{\cdot}6190, \quad \beta_2 = 6{\cdot}7905.$$