the latent heat h_{fg}, after making the two preceding assumptions is not warranted.

(4) Energy Balance

$$G_a dh = \pm G_L c_L dt_L \quad (39)$$

A minus sign refers to parallel flow of air and water and a plus sign to counterflow. Actually, the water flow rate changes between inlet and outlet due to the mass transfer. For exact energy balance the term ($c_L t_L dG_L$) should be added to the right side of Equation 39. The percentage change in G_L is quite small in all normal applications of air-conditioning equipment and, therefore, may be neglected.

(5) Heat Transfer to the Water

$$G_L c_L dt_L = h_L a_H (t_L - t_i) dl \quad (40)$$

Equations 36 to 40 are the basic relations necessary to the solution of simultaneous heat and mass transfer processes in direct contact air-conditioning equipment.

To facilitate the use of these relations in equipment design or performance, three other equations may be extracted from the above set. Combination of Equations 38, 39, and 40 gives

$$\frac{h - h_i}{t_L - t_i} = -\frac{h_L a_H}{K_D a_M} = -\frac{h_L}{K_D} \quad (41)$$

Equation 41 relates the enthalpy potential for the total heat transfer through the gas film to the temperature potential for this same transfer through the liquid film. As would be expected from physical reasoning, this ratio is proportional to the ratio of gas film resistance ($1/K_D$) to the liquid film resistance ($1/h_L$). Combination of Equations 37, 38, and the Lewis relation (Equation 32) gives

$$\frac{dh}{dt_a} = \frac{h - h_i}{t_a - t_i} \quad (42)$$

Similarly, combination of Equations 36, 37, and the Lewis relation gives

$$\frac{dW}{dt_a} = \frac{W - W_i}{t_a - t_i} \quad (43)$$

Equation 43 indicates that at any cross section in the spray chamber, the instantaneous slope of the air path, dW/dt_a, on a psychrometric chart is determined by a straight line connecting the air state with the interface saturation state at that cross-section. Thus in Fig. 12, state 1 represents the state of the air entering the parallel-flow air washer chamber

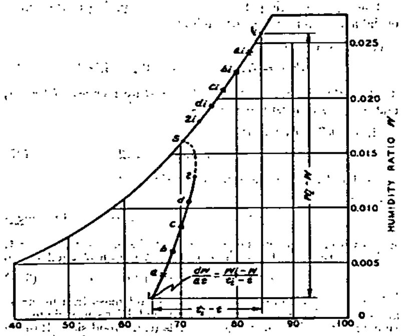

Fig. 12 .... Air Washer Humidification Process on Psychrometric Chart

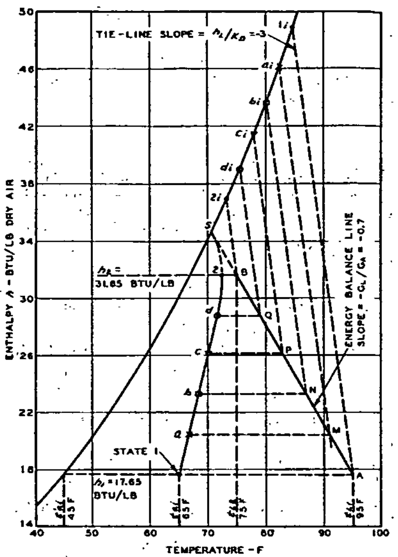

Fig. 13 .... Graphical Solution for the Air-State Path in a Parallel-Flow Air Washer

illustrated in Fig. 11. The washer is operating as a heating and humidifying apparatus so that the interface saturation state of the water at air inlet is the state designated 1_i. The initial slope of the air path is, then, along a line directed from state 1 to state 1_i. As the air is heated, the water cools and the interface temperature drops. Corresponding air states and interface saturation states are indicated by the letters a, b, c, d in Fig. 12. Note that in each instance the air path is directed toward the associated interface state. The interface states are found from Equations 39 and 41. Equation 39 describes how the air enthalpy changes with water temperature; and Equation 41, how the interface saturation state changes to accommodate this change in air and water conditions. The solution for the interface state on the normal psychrometric chart of Fig. 12 must be either by trial and error from Equations 39 and 41 or by a rather complex graphical procedure.[11] A much simpler method of solution results by using the method of Merkel[12] with improvements by Mickley.[13] This method utilizes a psychrometric chart with enthalpy and temperature as the coordinates. It is best illustrated by example.

Example 3. A parallel flow air washer is to be designed using a spray chamber like that illustrated in Fig. 11. The design conditions are as follows:

Water temperature at inlet, t_{L1} = 95 F
Water temperature at outlet, t_{L2} = 75 F
Air temperature at inlet, t_{a1} = 65 F
Air wet-bulb at inlet, t_{a1}' = 45 F
Air mass flow rate per unit area, G_a = 1200 lb per (hr) (sq ft)
Spray ratio, G_L/G_a = 0.70

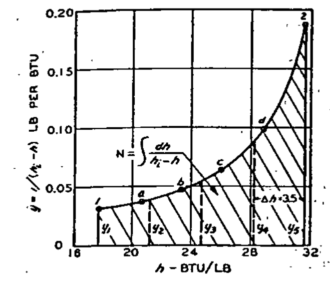

Fig. 14 .... Graphical Evaluation of $\int dh/(h_i - h)$

Air heat transfer coefficient per cubic foot of chamber volume, $h_a a_H$ = 72 Btu per (hr) (F deg) (cu ft)
Liquid heat transfer coefficient per cubic foot of chamber volume, $h_L a_H$ = 900 Btu per (hr) (F deg) (cu ft)
Air volume flow rate, Q = 6500 cfm

Solution: The air mass flow rate w_a = (6500/13.25) = 490 lb/min, and the required spray chamber cross-sectional area is, then $A_{cs} = w_a/G_a$ = 490 (60)/1200 = 24.5 sq ft. The mass transfer coefficient is given by the Lewis relation as

$$K_D a_M = (h_a a_H)/c_{pm} = 72/0.24 = 300 \text{ lb per (hr)(cu ft)}$$

Fig. 13 shows the enthalpy-temperature psychrometric chart with the graphical solution for the interface states and the air path through the washer spray chamber. The solution proceeds as follows:

1. Enter bottom of chart with t_{a1}' of 45 F, follow up to saturation curve to establish air enthalpy h_1 of 17.65 Btu/lb. Extend this enthalpy line to intersect initial air temperature t_{a1} of 65 F (state 1 of air), and initial water temperature t_{L1} of 95 F at point A. (Note carefully here that we utilize the temperature scale both for air and water temperatures.)

2. Through point A construct the *energy balance line* AB with a slope of

$$(dh/dt_L) = -G_L/G_a = -0.7$$

Point B is determined by intersection with the leaving water temperature t_{L2} = 75 F. The negative slope here is a consequence of the parallel flow, which results in the air-water mixture approaching, but not reaching, the common saturation state *s*. (The line AB has no physical significance as far as representing any *air state* on the psychrometric chart. It is merely used as a construction line in the graphical solution.)

3. Through point A construct the *tie-line* A1_i having a slope of

$$\frac{h - h_i}{t_L - t_i} = -\frac{h_L a_H}{K_D a_M} = -\frac{900}{300} = -3$$

The intersection of this line with the saturation curve gives the initial interface state 1_i at the chamber inlet. (Note carefully here how the energy balance line and tie-line, representing Equations 39 and 41, are combined to give a simple graphical solution on the $h - t$ chart for the interface state.)

4. The initial slope of the air path may now be constructed, according to Equation 42, by drawing line 1a toward the initial interface state 1_i. (The length of the line 1a will depend upon the degree of accuracy required in the solution and the rate at which the slope of the air path is changing.)

5. Construct the horizontal line a-M locating the point M on the energy-balance line. Draw a new tie-line (slope of −3 as before) from M to a_i locating interface state a_i. Continue the air path from a to b by directing it toward the new interface state a_i. (Note that the change in slope of the air path from 1a to ab is quite small, justifying the path incremental lengths used.)

6. Continue in the manner of step 5 until point 2, the final state of the air leaving the chamber is reached. In this example six steps are used in the graphical construction with the following results:

State	1	a	b	c	d	2
t_L	95.0	91.0	87.0	83.0	79.0	75.0
h	17.65	20.45	23.25	26.05	28.85	31.65
t_i	84.5	82.3	80.1	77.8	75.6	73.2
h_i	49.00	46.25	43.80	41.50	39.10	37.00
t_a	65.0	66.8	68.5	70.0	71.4	72.4

The final state of the air leaving the washer is t_{a2} = 72.4 F and h_2 = 31.65 Btu per lb (wet-bulb temperature t_{a2}' = 67.0 F).

7. The final step is to calculate the required length of the spray chamber. From Equation 38b

$$l = \frac{G_a}{K_D a_M} \int_1^2 \frac{dh}{(h_i - h)}$$

The integral is evaluated graphically by plotting $1/(h_i - h)$ *vs* h as shown in Fig. 14. Any satisfactory graphical method may be used to evaluate the area under the curve. Using Simpson's rule with four equal increments of Δh equal to 3.5 gives:

$$N = \int_1^2 \frac{dh}{(h_i - h)} \approx (\Delta h/3)(y_1 + 4y_2 + 2y_3 + 4y_4 + y_5)$$

$$N = (3.5/3)[(1 \times 0.0319) + (4 \times 0.0400) + (2 \times 0.0553) + (4 \times 0.0865) + (1 \times 0.1870)] = 0.975$$

The design length is, therefore, l = (1200/300)(0.975) = 3.9 ft.

The method illustrated by *Example 3* may be also used to predict the performance of existing direct-contact equipment. Furthermore, the method can serve the very important function of determining the transfer coefficients when performance data from test runs are available. That is, knowing the water and air temperatures entering and leaving the chamber and the spray ratio, it is possible by a trial and error procedure to determine the proper slope of the tie-line necessary to achieve the measured final air state. The tie-line slope gives the ratio $h_L a_H / K_D a_M$. Furthermore, $K_D a_M$ is determined from the integral relationship in *Example 3* from the known chamber length l.

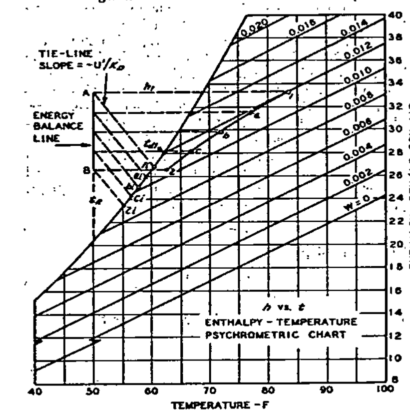

Fig. 15 .... Graphical Solution for the Air-State Path in a Direct-Expansion Coil Dehumidifier