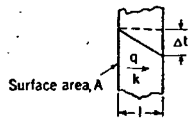
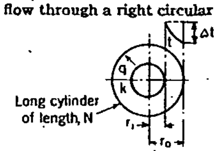
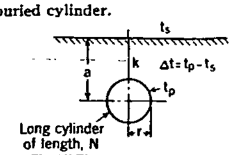
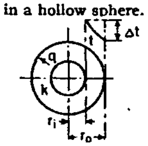
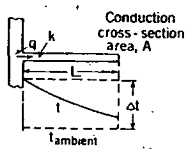
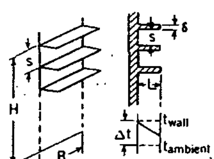
TABLE 8.—SOLUTIONS FOR SOME STEADY-STATE THERMAL CONDUCTION PROBLEMS[a,b]

No.	SYSTEM	Expressions for the resistance R entering into the equation: $q = \Delta t/R$ (Btu per hour)
1.	Flat wall or curved wall if curvature is small (wall thickness less than 0.1 of inside diameter). 	$R = \frac{L}{kA}$
2.	Radial flow through a right circular cylinder. 	$R = \frac{\log_e \frac{r_o}{r_i}}{2\pi kN}$ (See footnote c).
3.	The buried cylinder. 	$R = \frac{\log_e \frac{2a}{r}}{2\pi kN}$; $R = \frac{\cosh^{-1} \frac{a}{r}}{2\pi kN}$ for $\frac{a}{r} \geq 3$ (See footnote c).
4.	Radial flow in a hollow sphere. 	$R = \frac{\frac{1}{r_i} - \frac{1}{r_o}}{4\pi k}$
5.	The straight fin or rod heated at one end. 	$R = \frac{m}{h_s p \tanh m L}$ (see footnotes d and e). For $ml > 2.3$, $\tanh m L \approx 1$; $m = \sqrt{h_s p/kA}$; A = conduction cross-section area. p = perimeter of cross-section A. h_s = unit conductance to the surroundings from the fin surface. k = thermal conductivity fin material. Δt = wall temperature—ambient temperature
6.	Finned surface of area HB. 	$R = \frac{(s + \delta)}{h_s \left(\frac{2}{m} \tanh m\, l + s\right) HB}$; $m = \sqrt{\frac{h_s p}{kA}} = \sqrt{\frac{2\, h_s}{k\delta}}$; Δt defined as in Case 5 above.

[a]The dimensions to be employed in these solutions are: length of dimension p, L, r = feet; units of k = Btu per hour per square foot per degree Fahrenheit for one foot thickness; units of h, Btu per hour per square foot per degree Fahrenheit; units of area, A = square feet.

[b]The thermal conductivity, k, in these solutions should be taken at the average material temperature (see Table 5).

[c]$\log_e x = 2.303 \log_{10} x$.

[d]This expression can also be employed as an approximation for tapered fins or of annular fins by employing average magnitudes of A and p.

[e]Tanh is the hyperbolic tangent.

can be obtained from this relation. Then the heat transfer current for the length of pipe (N, ft) can be established by the relation:

$$q_{rc} \text{ (Btu per hour)} = \frac{t_o - t_f}{R_T} \tag{10}$$

For a unit length of the pipe the heat transfer rate is:

$$\frac{q_{rc}}{N} \text{ (Btu per hour foot)} = \frac{(t_o - t_f)}{R_T N} \tag{11}$$

The temperature drop, Δt, through an individual resistance may then be calculated from the relation:

$$\Delta t = R\, q_{rc}$$

where R is the resistance in question.

The problem is now reduced to one of evaluating the individual resistances of the system. This entails suitable integration of the rate Equations 1, 2 and 3 to produce expressions of the form:

$$q = \frac{\Delta t}{R} \tag{12}$$

where q is the heat transfer rate, and Δt is the potential drop or temperature difference through the resistance R. Table 8 lists such solutions for six different conduction systems. Table 2 in Chapter 4 and Table 1 of this chapter indicate the magnitudes of the thermal conductivities, k, to be employed in the expressions of Table 8.

The solution applicable to the problem depicted in Fig. 4, for the calculation of R_2 and R_3, is case 2 in Table 8. Thus for a *1 ft length* of 2 in. nominal size pipe (I. D. = 2.067 in., O. D. = 2.375 in.) insulated with 1 in. of cork:

$$R_2 = \frac{\log_e \frac{1.188}{1.033}}{2\pi \times 26 \times 1} = 8.5 \times 10^{-4} \text{ hr degree Fahrenheit per Btu.}$$

$$R_3 = \frac{\log_e \frac{2.188}{1.188}}{2\pi \times 0.025 \times 1} = 3.9 \text{ hr degree Fahrenheit per Btu.}$$

The convection resistances to heat transfer from the pipe wall to the cold water, R_1, and from the air to the surface of the insulating material, R_c, are dependent on the flow conditions prevailing at these surfaces, and on the thermal properties of the fluids. The unit conductances for thermal convection, h, Btu per hour per square foot per degree Fahrenheit, have been determined by test for many flow systems. These data may be employed to predict the conductances for *similar flow systems*. Table 5 summarizes some empirical equations expressing such test results.

For the problem under consideration (Fig. 4) case 3 of Table 5 is applicable for the calculation of the cold water side convection resistance R_1. Corresponding to the water velocity of 5 fps, the mass velocity is:

$$G = 5 \text{ (ft per sec)} \times 62.4 \text{ (lb per cu ft)} \times 3600 \text{ (sec per hr)} = 11.2 \times 10^5 \text{ lb per hour per square foot.}$$

The inside diameter of the pipe D is $\frac{2.067}{12} = 0.1725$ ft.