

$$w = A_2 \sqrt{\frac{2kgp_1 p_2}{k-1} \left[\left(\frac{p_1}{p_2} \right)^{\frac{k-1}{k}} - 1 \right]} = A_2 \sqrt{\frac{2kgp_1 p_2}{k-1} \left(\frac{p_2}{p_1} \right)^{\frac{k+1}{k}} \left[\left(\frac{p_1}{p_2} \right)^{\frac{k-1}{k}} - 1 \right]} \quad (40)$$

If this is computed and the figures are plotted, the curved line (partly solid and partly broken) of Fig. 6 is found. The maximum value of $\frac{p_2}{p_1}$ may be computed by differentiating w with respect to p_2 and equating the result to zero. This operation produces the formula:

$$\frac{p_2}{p_1} = \left(\frac{2}{k+1} \right)^{\frac{k}{k-1}} \quad (41)$$

For air, with $k = 1.40$, $\frac{p_2}{p_1} = 0.53$.

Actually, the broken part of the curve is not attained for the flow in the nozzle. If the ratio of p_2 to p_1 is decreased from unity, the mass

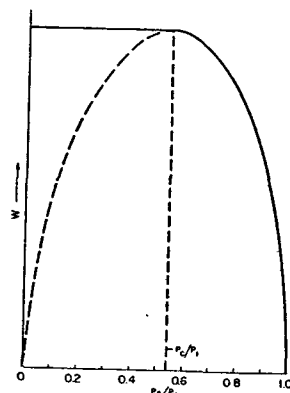


FIG. 6. RELATION OF FLOW OF GAS TO PRESSURE DROP IN A CONVERGING TUBE

rate of discharge, as well as the volume, increases from zero to a maximum, as shown by the solid section of the curve in Fig. 6; thereafter, as p_2/p_1 is decreased further, the discharge is constant, as indicated by the horizontal line. The value of p_2 at the maximum point is called the critical pressure, or p_c , and it is seen that p_c is approximately 53 per cent of p_1 when air is flowing.

To find the velocity at the critical pressure, it is assumed that the upstream velocity V_1 is so small as to be negligible. Using the subscript c to indicate conditions at the critical point, from Equation 31

$$\frac{p_c}{p_1} = \frac{1}{\left(1 + \frac{k-1}{2} M_c^2 \right)^{\frac{k}{k-1}}} \quad (42)$$

or

$$= \sqrt{\frac{2}{k-1} \left[\left(\frac{p_1}{p_c} \right)^{\frac{k-1}{k}} - 1 \right]} \quad (43)$$

Substituting the critical pressure ratio from Equation 41 it follows that

$$M_c = 1 \quad (44)$$

or that the velocity at the throat is equal to the local sonic velocity.

In developing the working equations for orifices and nozzles, it is customary to start with the incompressible form of the flow Equation 39. In this case both M_1 and M_2 are small quantities, $\rho_1 = \rho_2$, and $(p_1 - p_2)/p_1 = \Delta p/p_1$ is small. Retaining only first order terms, it follows from Equation 36 that

$$\frac{M_1}{M_2} = \frac{A_2}{A_1} \quad (45)$$

so that

$$\sqrt{\frac{1 + \frac{k-1}{2} M_1^2}{1 - M_1^2/M_2^2}} \approx \frac{1}{\sqrt{1 - (A_2/A_1)^2}} = \frac{1}{\sqrt{1 - \beta^2}} \quad (46)$$

where $\beta = D_2/D_1$. The quantity $1/\sqrt{1 - \beta^2}$ is the velocity of approach factor as generally used, with β being the ratio of the throat or orifice diameter to the pipe diameter.

Since $\Delta p/p_1$ is small

$$\frac{2k}{k-1} \left[\left(\frac{p_1}{p_2} \right)^{\frac{k-1}{k}} - 1 \right] \approx \frac{2\Delta p}{p_1} \quad (47)$$

and the mass flow is

$$w = \frac{A_2}{\sqrt{1 - \beta^2}} \sqrt{2g\rho\Delta p} \quad (48)$$

The volume flow is then

$$Q_v = A_2 \frac{1}{\sqrt{1 - \beta^2}} \sqrt{2g\Delta p/\rho} = A_2 \frac{1}{\sqrt{1 - \beta^2}} \sqrt{2gh} \quad (49)$$

Actual Flow Through Orifices and Nozzles

The actual rate of flow through an orifice, nozzle, or Venturi tube is rarely equal to the theoretical, and generally the actual rate is less than the theoretical. In the case of the nozzle and Venturi tube, this is due to losses from wall roughness, fluid friction, and turbulence during the expansion in the section following the throat. While wall roughness is not a factor in a sharp-edged orifice, fluid friction and turbulence are important, as is the fact that the discharge contracts to a degree variable with the ratio of outlet to inlet pressure after leaving the orifice, so that the limiting area is somewhat less than the opening in the orifice plate. Accordingly, Equation 49 must be modified by a correction factor, C . Usually, the velocity of approach factor is included with this correction factor, and, if

$$K = C \frac{1}{\sqrt{1 - \beta^2}} \quad (50)$$

$$Q_v = K A_2 \sqrt{2gh} \quad (51)$$

Multiplying by 3600 to convert from cubic feet per second to cubic feet