Note the analogy to the direct current electrical circuit problem. A temperature (potential) drop is required to overcome these resistances to the flow of thermal current. The total resistance to heat transfer, R_T, hour Fahrenheit degrees per Btu, is the summation of the individual resistances:

$$R_T = R_1 + R_2 + R_3 + R_4 \tag{9}$$

where the resultant parallel resistance R_4 is obtained from:

$$\frac{1}{R_4} = \frac{1}{R_r} + \frac{1}{R_c}$$

Provided the individual resistances may be evaluated, the total resistance can be obtained from this relation. Then the heat transfer for the length of pipe (N, ft) can be established by the relation:

$$q_{re}\ (\text{Btu per hour}) = \frac{(t_o - t_i)}{R_T} \tag{10}$$

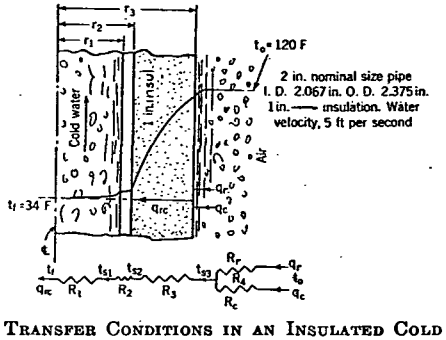


FIG. 4. HEAT TRANSFER CONDITIONS IN AN INSULATED COLD WATER LINE

For a unit length of the pipe the heat transfer rate is:

$$\frac{q_{re}}{N}\ (\text{Btu per hour foot}) = \frac{(t_o - t_i)}{R_T N} \tag{11}$$

The temperature drop, Δt, through an individual resistance may then be calculated from the relation:

$$\Delta t = R\, q_{re}$$

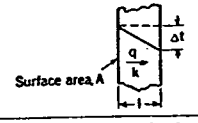
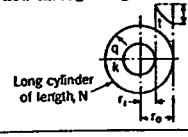
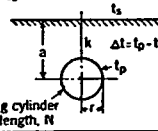
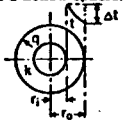
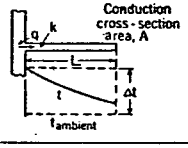
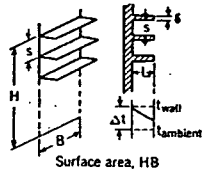
where R is the resistance in question.

The problem is now reduced to one of evaluating the individual resistances of the system. This entails suitable integration of the rate Equations 1, 2 and 3 to produce expressions of the form:

$$q = \frac{\Delta t}{R} \tag{12}$$

where q is the heat transfer rate, and Δt is the potential drop or temperature difference through the resistance R. Table 6 lists such solutions for six different conduction systems. Table 2 in Chapter 9 and Table 1 of

TABLE 6. SOLUTIONS FOR SOME STEADY-STATE THERMAL CONDUCTION PROBLEMS[a, b]

No.	System	Expressions for the resistance R entering into the equation: $q = \Delta t/R$ (Btu per hour)
1.	Flat wall or curved wall if curvature is small (wall thickness less than 0.1 of inside diameter). 	$R = \frac{L}{kA}$
2.	Radial flow through a right circular cylinder. 	$R = \frac{\log_e \frac{r_o}{r_i}}{2\pi kN}$ (See footnote c).
3.	The buried cylinder. 	$R = \frac{\log_e\left(\frac{a+\sqrt{a^2+r^2}}{r}\right)}{2\pi kN} - \frac{\cosh^{-1}\left(\frac{a}{r}\sqrt{1+\frac{r^2}{a^2}}\right)}{2\pi kN}$ For $\frac{a}{r} \geq 3$, a satisfactory approximation is: $R = \frac{\log_e \frac{2a}{r}}{2\pi kN} - \frac{\cosh^{-1}\frac{a}{r}}{2\pi kN}$
4.	Radial flow in a hollow sphere. 	$R = \frac{\frac{1}{r_i} - \frac{1}{r_o}}{4\pi k}$
5.	The straight fin or rod heated at one end. 	$R = \frac{m}{h_o p \tanh mL}$ (see footnotes d and e). For $ml > 2.3$, $\tanh mL \approx 1$ $m = \sqrt{h_o p/kA}$ A = conduction cross-section area. p = perimeter of cross-section A. h_o = unit conductance to the surroundings from the fin surface. k = thermal conductivity fin material. Δt = wall temperature—ambient temperature
6.	Finned surface of area HB. 	$R = \frac{(s+\delta)}{h_o\left(\frac{2}{m}\tanh ml + s\right)HB}$ $m = \sqrt{\frac{h_o p}{kA}} = \sqrt{\frac{2h_o}{k\delta}}$ Δt defined as in Case 5 above.

[a] The dimensions to be employed in these solutions are: length of dimension p, L, r = feet; units of k = Btu per (hour) (square foot) (Fahrenheit degree for one foot thickness); units of h, Btu per (hour) (square foot) (Fahrenheit degree); units of area, A = square feet.

[b] The thermal conductivity, k, in these solutions should be taken at the average material temperature (see Table 2).

[c] $\text{Log}_e x = 2.303 \log_{10} x$.

[d] This expression can also be employed as an approximation for tapered fins or of annular fins by employing average magnitudes of A and p.

[e] tanh is the hyperbolic tangent.