Replacing $v$ by its equal $g/g_c\rho$ (where $\rho$ is density in pounds weight per cubic foot) and rearranging, Equation 3 becomes

$$\frac{1}{2g}\,dV^2 + \frac{1}{\rho}\,dp + dz + \frac{g_c}{g}\,[J\,du + p\,dv - J\,dq + dW] = 0 \qquad (4)$$

In the case of flow through a pipe, no outside work is performed so that $dW = 0$. Furthermore,

$$J\,du + p\,dv = JT\,ds = J\,dq + JT\,ds' \qquad (5)$$

*where*

$ds$ = total change in entropy.

$ds'$ = change in entropy due to internal irreversibility from turbulence and friction.

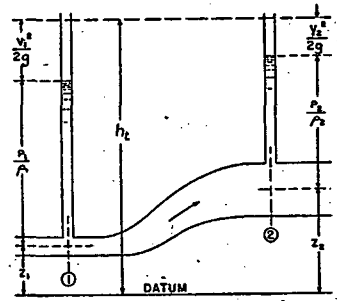


FIG. 1. RELATION OF VARIOUS FACTORS IN BERNOULLI EQUATION

Accordingly, Equation 4 may be written

$$\frac{1}{2g}\,dV^2 + \frac{dp}{\rho} + dz + \frac{g_c}{g}\,JT\,ds' = 0^* \qquad (6)$$

In cases where there is no internal irreversibility, $ds' = 0$, and Equation 6 may be integrated to give

$$\frac{V_1^2}{2g} + \frac{p_1}{\rho_m} + z_1 = \frac{V_2^2}{2g} + \frac{p_2}{\rho_m} + z_2 \qquad (7)$$

where $\rho_m$ is the proper mean density.

This is commonly called the Bernoulli equation, named after the Swiss mathematician and physician who first propounded the theory. $\frac{V^2}{2g}$ is known as the velocity head, $\frac{p}{\rho}$ is the pressure head, and $z$ is the elevation head, all in feet of the fluid; the total head, $h_t$ is the sum of the other three heads. Fig. 1 shows diagrammatically the relation of the various factors. The pressure at point 2 is lower than at point 1 because of the elevation of point 2 over point 1, and the velocity at point 2 is lower than at point 1 because of the larger pipe diameter at point 2. If the pipe

* In the analysis of subsequent portions of this chapter the distinction between $g$ and $g_c$ will be omitted. Aside from the dimensional consistency the factor, $g/g_c$, is not in general significant in fluid flow analysis.



diameter were the same throughout, the velocity, and consequently the velocity head, would be the same at both points, but the higher elevation at point 2 would still be responsible for a loss in pressure. The utility of the equation is evident, though it should be remembered that in it the *effects of friction and turbulence are neglected*, and that Fig. 1 represents ideal conditions. It should also be noted that care must be taken in determining the proper mean density. Accordingly, the Bernoulli equation is applied most conveniently to incompressible fluids for which density is constant.

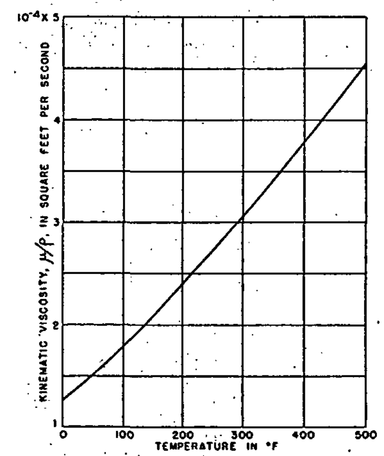


FIG. 2. RELATION OF KINEMATIC VISCOSITY TO TEMPERATURE OF AIR

## Pressure Loss in Circular Pipes

The pressure loss in circular pipes is customarily expressed by the formula:

$$h_f = \frac{flV^2}{2g\,d} \qquad (8)$$

*where*

$h_f$ = the loss in head of the fluid under conditions of flow, in feet.
$l$ = the length of the pipe, in feet.
$V$ = the velocity, in feet per second.
$g$ = the acceleration due to gravity = 32.174 ft per (second) (second).
d = the internal diameter of the pipe, in feet.
$f$ = a dimensionless friction coefficient.

The formula is generally known by the name of Darcy or Fanning, though it seems to have been originated by d'Aubisson de Voisins in 1834.

The factor $f$ is a function of the Reynolds number,

$$N_{Re} = \frac{d\,V\rho}{\mu} \qquad (9)$$