

compared to the absolute temperature T , then the mass transfer coefficient h_D will have an identical magnitude in Equations 2a, 2b, and 2c. As the property variations increase, however, the value of the coefficients will diverge.

Since the water vapor transferred to the air stream must be supplied with its latent heat of vaporization (h_{fg}), the heat flux between the liquid and the interface (and, consequently, the heat supplied by the heater) is

$$q/A = q_c/A + (\dot{m}_w/A)h_{fg}$$

or

$$q/A = h_L(t_s - t_w) + h_D(\rho_{w,s} - \rho_{w,\infty})h_{fg} \quad (3)$$

Furthermore

$$q/A = h_L(t_s - t_w) \quad (4)$$

where

q = heat transfer rate between liquid and interface, Btu per hour.

h_{fg} = enthalpy of evaporation (latent heat of vaporization), Btu per pound.

h_L = heat transfer coefficient for the liquid side, Btu per (hour) (square foot) (Fahrenheit degree).

If the flow conditions are specified so that the heat and mass transfer coefficients can be predicted, and the air stream properties and pan water temperature are given, Equations 1, 2, 3, and 4 can be solved simultaneously to find: (1) the interface saturation state, (2) the mass transfer rate, (3) the heat transfer to the air, and (4) the total heat transfer supplied to the water by the heater.

In the preceding discussion, the temperatures of the air stream and the water in the pan remained constant throughout the entire process. However, in many processes, such as in the countercurrent cooling tower, the air and water temperatures vary throughout the process, with resultant variations in the interface state. Equations 1, 2, 3, and 4 can be used to determine the total transfer in a step-by-step procedure, solving in each step for the interface state and the transfer rates, and then summing or integrating over each step to obtain the total transfer.

However, a simpler method is available and is more commonly used. Before developing this method, it is desirable to discuss the two means by which mass transfer or diffusion may occur: (1) molecular diffusion, and (2) convective or eddy diffusion.

MOLECULAR DIFFUSION

Consider a quantity of air in a small closed container where the air at one end has been slightly heated to cause a temperature gradient in the air. If the container is isolated, the temperature will soon equalize as a result of the passing of energy by a random mixing of the air molecules. This is the familiar process of heat conduction in a gas. Assume that a small amount of water vapor is introduced at one end of the container which is again isolated. After some time, the water vapor will be equally distributed throughout the air by the same molecular movements. This mass transfer process is known as *molecular diffusion*. The equations governing this process are analogous to those for heat conduction.

Basic Equation

The basic equation for molecular diffusion is usually referred to as *Fick's law* and can be derived from the principles of the kinetic theory of gases. Depending upon the parameter chosen for the driving potential, the one-dimensional diffusion rate per unit area in the y direction (normal to the transfer surface) is expressed as

Table 1 Mass Diffusivities for Gases in Air
(Gases at 77 F and 1 atm)

Gas	D_a in sq ft per hr
Ammonia	1.08
Benzene	0.34
Carbon Dioxide	0.64
Ethanol	0.46
Hydrogen	1.60
Oxygen	0.80
Water Vapor	0.99

$$\dot{m}_x/A = -D_a(d\rho_x/dy) \quad (5a)$$

$$\dot{m}_x/A = -\rho D_a(d\omega_x/dy) \quad (5b)$$

$$\dot{m}_x/A = -(D_a/R.T)(d\rho_x/dy) \quad (5c)$$

The minus sign appears because the concentration gradient is negative in the direction of diffusion. The quantity D_a is known as the *mass diffusivity* or the *diffusion coefficient* and is a property of the gas mixture. Since the mass flux ($\dot{m}_x/A$) is expressed in pounds per (hour) (square foot) and the partial density (ρ_x) in pounds per cubic foot, in the English system of units, the units of D_a for Equation 5a must be

$$D_a = [\text{lb per (hr)}] / [(\text{ft}^3 \text{ per lb}) (\text{ft})] = \text{ft}^2/\text{hr}$$

The Diffusion Coefficient

Extensive gas kinetic calculations¹ show that D_a depends only slightly on the concentration of the components in the mixture, that it is inversely proportional to the pressure, and that with temperature its increase is proportional to $T^{1.75}$, where n has a value between 0.5 and 1. The equations for diffusion coefficients in gases, derived by several investigators from kinetic theory models, may all be expressed by Equation 6:

$$D_a = b \frac{T^{1.75}}{P_r(\nu_{MA}^{1/2} + \nu_{MB}^{1/2})^2} \sqrt{\frac{1}{M_A} + \frac{1}{M_B}} \quad (6)$$

where

b = a constant.

P_r = total pressure, atmospheres.

M_A, M_B = molecular weights of the two gases in the gas mixture, pounds per pound mol.

ν_{MA}, ν_{MB} = molecular volumes of the two gases in the mixture, cubic feet per pound mol. (See Reference 2 for a table to compute these values.)

Upon noting that several theories agreed on this form, Gilliland evaluated the constant b empirically by plotting the results of some 400 experimental determinations. The resulting value was

$$b = 0.0069$$

The principal weakness of Equation 6 is the temperature function $T^{1.75}$, since D_a is more nearly proportional to T^n . Where experimental data are available for D_a , they should be preferred to the semi-theoretical Equation 6. A few experimental values for diffusion of some common gases in air are given in Table 1. More extensive tables are available in References 2, 3, and 4.

Of particular interest to the air conditioning engineer is the mass diffusivity of water vapor in air. Analyzing the available data, Spalding found that, up to a temperature of 2000 F, the mass diffusivity of water vapor in air may be expressed as

$$D_a = \frac{0.000146}{P_r} \frac{T^{1.75}}{T + 441} \quad (7)$$

Mass Transfer

where all quantities are expressed in the same units as in Equation 6.

Analogy to Conduction Heat Transfer

Molecular diffusion is directly analogous to conduction heat transfer. Both phenomena are a result of random molecular mixing in a stagnant fluid, or in a fluid in laminar (streamline) flow. In each case the equation governing the process can be expressed in the form

$$\text{Flux} = \text{diffusivity} \times \text{concentration gradient}$$

For a two-component gaseous system (e.g., water vapor and air) in which the density of the gas ρ and the specific heat c_p are constant, Equations 8 and 9 may be written for the fluxes of mass and energy in one-dimensional systems:

$$\dot{m}_x/A = -D_a \frac{d}{dy} \left(\frac{\rho_x}{\rho} \right) \quad (\text{Fick's law for constant } \rho) \quad (8)$$

$$q/A = -k \frac{dT}{dy} \quad (T = -\alpha \frac{d}{dy} (c_p T)) \quad (\text{Fourier's law for constant } \rho c_p) \quad (9)$$

where

k = thermal conductivity, Btu per (hour) (square foot) (Fahrenheit degree per foot).

α = thermal diffusivity, square feet per hour.

c_p = specific heat at constant pressure, Btu per (pound) (Fahrenheit degree).

Equation 8 states that mass transfer occurs because there is a gradient in mass concentration, and the mass flux equals the mass diffusivity D_a times the mass concentration gradient. Equation 9 states that heat transfer occurs because there is a gradient in energy concentration, and the heat flux equals the thermal diffusivity $\alpha (=k/\rho c_p)$ times the energy concentration gradient.

The equation for the viscous shear force in one-dimensional flow is

$$\tau_{yx} = -\mu \frac{dv_x}{dy} \quad (V_x)$$

or

$$\tau_{yx} = -\mu \frac{d}{dy} (V_x) \quad (\text{Newton's law for constant } \mu) \quad (10)$$

where

τ_{yx} = viscous shear force in the yx plane, pounds force per square foot.

g = gravitational acceleration = 4.17×10^8 (pounds force) (feet) per (pound mass) (hour) (hour).

μ = absolute viscosity, pounds per foot-hour.

V_x = local velocity of gas in the x direction parallel to the transfer surface, feet per hour.

ν = kinematic viscosity, or momentum diffusivity, square feet per hour.

Equation 10 states that momentum transfer occurs because there is a gradient in momentum concentration, and the momentum flux (τ_{yx}) equals the *momentum diffusivity* (kinematic viscosity) ν times the momentum concentration gradient.

It is important to note that the mass, thermal, and momentum diffusivities all have the same units (ft^2/hr) and that the concentration gradients are linear for a uniform medium at steady state. If, under a fortunate circumstance, these diffusivities all had the same numerical value, the concentration gradients would be identical. That is, if a fluid were in steady one-dimensional laminar flow over a surface such that $D_a = \alpha = \nu$, the partial pressure, temperature, and velocity

profiles would all be identical in form. This is true if ρ and c_p are essentially constant through the fluid field and the diffusivities are identical. Stated in terms of dimensionless parameters the condition is

$$\frac{\nu}{\alpha} = \left(\frac{\mu}{\rho} \right) \left(\frac{\rho c_p}{k} \right) = \frac{c_p \mu}{k} = N_{Pr} = 1$$

$$\frac{\nu}{D_a} = \frac{\mu}{\rho D_a} = N_{Sc} = 1$$

where

$$N_{Pr} = \text{Prandtl number} = \frac{c_p \mu}{k}, \text{ dimensionless.}$$

$$N_{Sc} = \text{Schmidt number} = \frac{\mu}{\rho D_a}, \text{ dimensionless.}$$

Where the Prandtl number ($N_{Pr} = c_p \mu/k$) equals the Schmidt number ($N_{Sc} = \mu/\rho D_a$), the temperature and partial pressure profiles are identical; and if, in addition, both are unity, these

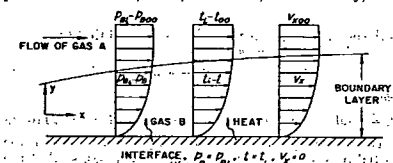


Fig. 2 Laminar Boundary Layer on a Flat Surface

profiles are the same as the velocity profile. This condition is illustrated in Fig. 2 for a gas B diffusing from a stationary boundary into a gas A in one-dimensional steady flow past the boundary. For air and water vapor at normal room conditions the Prandtl number is 0.71 and the Schmidt number is 0.60, a condition surprisingly close to that postulated.

Equimolar Counterdiffusion

Consider a mixture of air and water vapor in a closed container in equilibrium at all points throughout the container so that the total pressure (in pounds force per square foot) is P_t and the absolute temperature T . A concentration gradient exists through the mixture so that the partial pressure p_v of the water vapor is as shown in Fig. 3. It is assumed here that

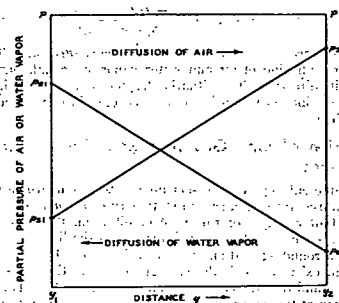


Fig. 3 Partial Pressure Profiles in Equimolar Counter-diffusion