the Swiss mathematician and physicist who first propounded the theory. $\frac{V^2}{2g}$ is known as the velocity head, $\frac{p}{\rho}$ is the pressure head, and z is the elevation head, all in feet of the fluid; the total head, h_t is the sum of the other three heads. Fig. 1 shows diagrammatically the relation of the various factors. The pressure at point 2 is lower than at point 1 because of the elevation of point 2 over point 1, and the velocity at point 2 is lower than at point 1 because of the larger pipe diameter at point 2. If the pipe diameter were the same throughout, the velocity, and consequently the velocity head, would be the same at both points, but the higher elevation at point 2 would still be responsible for a loss in pressure. The utility of the equation is evident, though it should be remembered that in it the *effects of friction and turbulence are neglected*, and that Fig. 1 represents ideal conditions. It should also be noted that care must be taken in determining the proper mean density. Accordingly, the Bernoulli equation is applied most conveniently to incompressible fluids for which density is constant.

Pressure Loss in Circular Pipes

The pressure loss in circular pipes is customarily expressed by the formula:

$$h_f = f \cdot \frac{l}{d} \cdot \frac{V^2}{2g} \tag{8}$$

where

- h_f = the loss in head of the fluid under conditions of flow, feet.
- l = the length of the pipe, feet.
- V = the velocity, feet per second.
- g = the acceleration due to gravity = 32.174 ft per (second) (second).
- d = the internal diameter of the pipe, feet.
- f = a dimensionless friction coefficient.

The formula is generally known by the name of Darcy or Fanning.

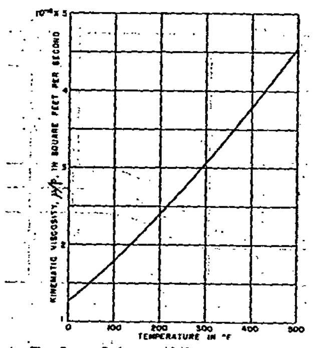


Fig. 2 .... Relation of Kinematic Viscosity to Temperature of Air

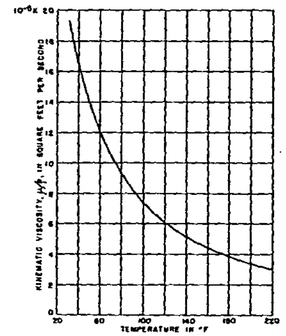


Fig. 3 .... Relation of Kinematic Viscosity to Temperature of Water

The factor f is a function of the Reynolds number,

$$N_{Re} = \frac{d\,V\rho}{\mu} \tag{9}$$

where

- N_{Re} = Reynolds number.
- ρ = the density, pounds per cubic foot.
- μ = the absolute viscosity, pounds per foot-second.

Both f and the Reynolds number are dimensionless. To aid in computing the Reynolds number, values of $\frac{\mu}{\rho}$, the kinematic viscosity, are shown as a function of temperature for air in Fig. 2, and for water in Fig. 3.

Fig. 4 shows the relation between f and the Reynolds number, adapted from a review by Moody.[1] The straight line sloping downward at the left of the chart supplies the values of f for laminar flow determined by the formula:

$$f = \frac{64}{N_{Re}} \tag{10}$$

With laminar flow, the velocity profile is a parabola, having the formula:

$$V = \frac{\rho h_f}{4\mu l}(r^2 - L^2) \tag{11}$$

where

- r = the radius of the pipe, feet.
- L = distance perpendicularly from the axis of the pipe, feet.

Accordingly, the maximum velocity occurs at the center of the pipe and is twice the average velocity; the average velocity is found when L = 0.707 r. It is worth noting that roughness of the pipe wall has no effect on the loss in head for laminar flow.

Between values of the Reynolds number of 2000 and 4000,

[1] Superior numbers refer to the references at the end of chapter.

there is an unstable region where the flow changes from laminar to turbulent, or vice versa. The actual value is impossible of prediction for any conditions of flow, though in general it may be said that the prevailing type of flow persists into the unstable region; however, once a change starts, it proceeds very rapidly.

When the flow is turbulent, the velocity profile is essentially parabolic over four-fifths of the pipe diameter, but near the pipe walls, the effect of friction becomes evident, and in the boundary layer at the pipe wall the flow is laminar. Fig. 5 compares the velocity profiles for three different Reynolds numbers, but for the same average velocity.

The lower curve in the turbulent region in Fig. 4 represents the relation of f to the Reynolds number for smooth pipe, such as drawn brass tubing or glass tubing. The effect of roughness on f, an effect which is considerable in turbulent flow, is open to some conjecture; artificially roughened pipes, for instance, give results at variance with actual tests. The curves above the smooth pipe curve of Fig. 4 represent a summary of tests on rough pipe, each of them identified by a value of e/d, with e signifying the absolute roughness in feet. Values of e for different pipes are given in Table 1.

To find the friction loss for any pipe, follow the curve with the proper value of e/d, to the pertinent value of N_{Re}, and from this point proceed horizontally to left margin to find the value of f for use in Equation 8.

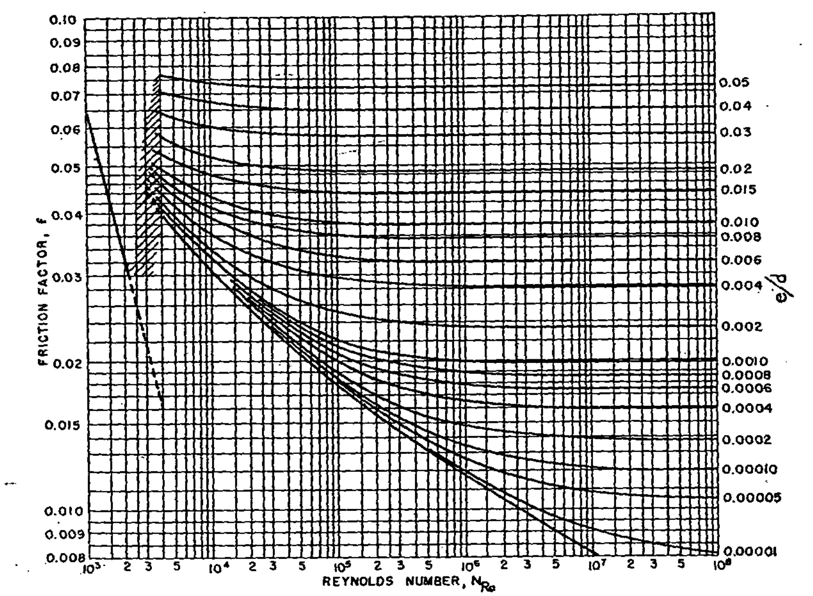
The curves in Fig. 4 may be approximated very closely by the empirical formula:[1]

$$f = 0.0055\left[1 + \left(20{,}000\,\frac{e}{d} + \frac{10^6}{N_{Re}}\right)^{1/3}\right] \tag{12}$$

Equation 8 is applicable to all liquids, and to gases when

Table 1 .... Values of e for Different Kinds of Pipe

Type of Pipe	e
Smooth drawn tubing	0.000005
Commercial steel or wrought iron	0.00015
Asphalted cast-iron	0.0004
Galvanized iron	0.0005
Cast-iron	0.00085
Wood stave	0.0006 to 0.003
Concrete	0.001 to 0.01
Riveted steel	0.003 to 0.03



Note: The straight line at left shows values of Friction Factor for laminar flow. *Reprinted by permission from ASME Transactions.*

Fig. 4 .... Relation Between Friction Factor and Reynolds Number