or, by rearrangement,

$$\frac{p_2}{p_1} = \left[1 - \frac{k-1}{V_{s0}^2}\left(\frac{V_2^2 - V_1^2}{2}\right)\right]^{\frac{k}{k-1}} \qquad (27)$$

which permits the calculation of the ratio of pressures at entrance and exit of the steady flow device,—pipe, orifice, or nozzle.

FLOW THROUGH NOZZLE OR ORIFICE

Another useful expression, covering the energy change in an orifice or nozzle, may be derived from Equation 2. As with the flow through pipes, no outside work is done. Then, assuming that there is no difference in elevation, and since practically no heat is evolved or absorbed, *i.e.*, the process is adiabatic, E, z, and q of Equation 2 may be eliminated, and, by rearranging, the equation becomes

$$\frac{V_2^2}{2g} - \frac{V_1^2}{2g} = J(h_1 - h_2) \text{ foot pounds per second} \qquad (28)$$

In any flow device,

$$\frac{V_1A_1}{v_1} = \frac{V_2A_2}{v_2} \text{ or, } V_1 = V_2\frac{A_2v_1}{A_1v_2} \qquad (29)$$

in which A_1 or A_2 is the cross-sectional area of the flow at a particular point, expressed in square feet. With this substituted in Equation 28, and solving for V_2:

$$V_2 = \sqrt{\frac{2gJ(h_1 - h_2)}{1 - (A_2/A_1)^2\,(v_1/v_2)^2}} \qquad (30)$$

Using this expression, it is possible to determine the velocity at any point in the flow through an orifice or nozzle. If the area at the point of entry is very large with respect to that at point 2, the denominator on the right side of Equation 30 will approach unity, and the equation will reduce to

$$V_2 = \sqrt{2gJ(h_1 - h_2)} \qquad (31)$$

For this reason, the expression $\sqrt{\dfrac{1}{1 - (A_2/A_1)^2\,(v_1^2/v_2^2)}}$ is called the correction factor for the velocity of approach.

The velocity of approach factor may be further simplified if the difference in volume between points 1 and 2 is negligible. Under this condition, the velocity of approach factor becomes $\sqrt{\dfrac{1}{1 - (A_2/A_1)^2}}$. If

$$\frac{A_2}{A_1} = \frac{D_2^2}{D_1^2} = \beta^2 \qquad (32)$$

the velocity of approach factor is $\sqrt{\dfrac{1}{1 - \beta^4}}$, in which form it is generally used in flow formulas. The quantity β is the ratio of the throat or orifice diameter to the pipe diameter.

The connection of the velocity of sound with the flow of fluids has already been noted. Its most important application is to the flow of gases through a converging tube or nozzle. If it is assumed that the inlet velocity of the fluid, V_1, is negligible, Equation 24 will reduce to

$$V_2 = \sqrt{\left(\frac{2gk}{k-1}\right)\frac{p_1}{\rho_1}\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}\right]} \qquad (33)$$

Let W represent the weight of gas flowing through the converging tube in a unit of time, and A_2 the area at the throat; then,

$$W = A_2V_2/v_2 \text{ or } V_2 = Wv_2/A_2 \qquad (34)$$

Substituting this, as well as the relation $p_1v_1^k = p_2v_2^k$, in Equation 33 gives,

$$W = A_2\sqrt{\left(\frac{2gk}{k-1}\right)\frac{p_1}{v_1}\left[\left(\frac{p_2}{p_1}\right)^{\frac{2}{k}} - \left(\frac{p_2}{p_1}\right)^{\frac{k+1}{k}}\right]} \qquad (35)$$

If this is computed and the figures are plotted, the curved line (partly solid and partly broken) of Fig. 6 is found. The maximum value of $\frac{p_2}{p_1}$ may be computed by differentiating W with respect to p_2 and equating the result to zero. This operation produces the formula:

$$\frac{p_2}{p_1} = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}} \qquad (36)$$

For air, with $k = 1.40$, $\frac{p_2}{p_1} = 0.53$.

Critical Pressure and Critical Flow

Actually, the broken part of the curve is not attained for the flow in the nozzle. If the ratio of p_2 to p_1 is decreased from unity, the weight

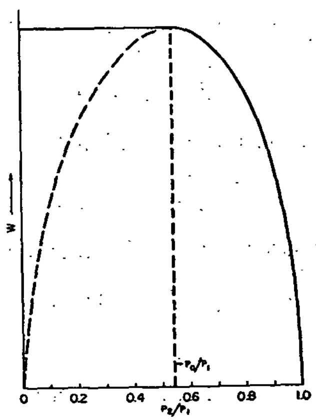

FIG. 6. RELATION OF FLOW OF GAS TO PRESSURE DROP IN A CONVERGING TUBE