The effectiveness, therefore, is independent of the temperatures in the exchanger. For any exchanger in which the capacity rate ratio Z is zero, the effectiveness is:

$$\eta = 1 - \exp(-\mathrm{NTU}) \tag{40}$$

For example, this equation applies where one of the fluids undergoes a change of phase in passing through the exchanger (e.g., condenser or evaporator).

The heat transferred can be determined from the energy equation:

$$q = C_h(t_{hi} - t_{ho}) = C_c(t_{co} - t_{ci}) \tag{41}$$

or the heat transferred can be determined from a form of the rate equation requiring only the entering fluid temperatures:

$$q = \eta C_{min}(t_{hi} - t_{ci}) \tag{42}$$

The proper mean temperature difference for the rate equation (Equation 34) then is given by:

$$\Delta t_m = \frac{\eta}{(\mathrm{NTU})}(t_{hi} - t_{ci}) \tag{43}$$

The effectiveness for parallel-flow exchangers is:

$$\eta = \frac{1 - \exp\left[-(\mathrm{NTU})(1+Z)\right]}{1+Z} \tag{44}$$

When, for a parallel-flow exchanger, $Z = 1$:

$$\eta_{Z=1} = \frac{1 - \exp(-2\,\mathrm{NTU})}{2} \tag{45}$$

The effectiveness for counter-flow exchangers is:

$$\eta = \frac{1 - \exp\left[-(\mathrm{NTU})(1-Z)\right]}{1 - Z\exp\left[-(\mathrm{NTU})(1-Z)\right]} \tag{46}$$

$$\eta_{Z=1} = \frac{(\mathrm{NTU})}{1 + (\mathrm{NTU})} \tag{47}$$

Graphical solutions to Equations 44 to 47, together with the more complicated cases of cross-flow, cross-counterflow, parallel-counterflow, and multi-pass counterflow, are presented by London and Kays.[85]

THERMAL INSULATION

Heat transfer through insulating materials can take place by conduction, convection or radiation. Ordinarily, all three modes are involved with one or two predominating. For a more detailed discussion of thermal insulation, see Chapters 22 and 24.

Vacuum

The use of vacuum as a means of insulation has been throughly reviewed by Scott.[96]

Vacuum is the most effective way of providing insulation because two of the principal modes of heat transfer, gaseous conduction and convection, are practically eliminated. Heat loss is principally through radiation, and conduction through solid supporting members. When the temperature difference is large (for example, in containers for storing liquid nitrogen at ambient temperatures) heat loss is almost entirely by radiation. When the temperature difference is small, conduction by residual gas may become important.

At atmospheric pressures and below, the thermal conductivity of a gas is independent of pressure. At very low pressures, however, where the mean free path of the molecules becomes significant compared with the distance between surfaces, the thermal conductivity decreases. At pressures below about 1 micron of mercury, the heat transfer rate is nearly proportional to the pressure.

Some generalizations about reflectors have been made by Scott.[96]

1. The best reflectors are also the best electrical conductors (copper, silver, gold, aluminum).
2. The emissivity decreases with decreasing temperature.
3. The emissivity of good reflectors is increased by surface contamination.
4. Alloying a good-reflecting metal increases its emissivity.
5. The emissivity is increased by treatments such as mechanical polishing which results in work-hardening of the surface layer of metal.
6. Visual appearance (i.e., brightness) is not a reliable criterion of reflecting power at long wave lengths.

Evacuated Porous Insulation

The presence of powder or fibers in an evacuated system reduces heat transfer by radiation and convection and so provides slightly better insulation than vacuum alone.[97] With finely divided powders, where the distance between particles is less than the mean free path of the gas, this improvement may not be observed.

Porous Insulation

A general discussion of the mechanism of heat transfer in porous insulation has been published by Stephenson and Mark.[98] Porous insulation is defined as any material containing gaseous pores such as fibers, textiles, granules, foams, etc. Heat transfer through these materials is rather complex and depends on thermal conductivity of solid and gas, cell size and shape, apparent density and other factors.

Conduction through solid material and gaseous pores is the principal method of heat transfer but convection and radiation may also be important. An individual analysis for each insulation is necessary.

The effective heat transfer of a cellular material usually increases with increasing apparent density, but is also affected by the size and number of gas cells present. Radiant heat flow is decreased as the number of cells is increased, since heat is absorbed and re-radiated at each intervening cell wall. Heat can be transmitted by gaseous convection within a cell and this effect is directly related to cell size. For a detailed treatment of these relationships, see Reference 98.

The nature of the gas also influences the effective insulation value of porous materials.[99–101] The presence of high-molecular weight gas in the cells increases the insulating effect, especially when circulation, directness and solid conduction parameters are all small. The results of a theoretical analysis[99] of several types of insulation are listed in Table 10. The effect of a heavy gas can be seen by a comparison of the values for fibrous glass.

Measured values of the conductivity of several insulating materials are given in Table 11. Again, the effect of a heavy gas (Refrigerant 11) in reducing the conductivity can be seen by a comparison of the values for urethane foam.

Table 10 .... Estimated Conductivity of Various Insulation Materials[99]

Mean Temp. = 65 F. Air Velocity 50 fpm.

Insulation	k, Btu/(hr) (sq ft) (F deg per in.)
Fibrous glass supported vacuum	0.04 to 0.06
Heavy gas-filled fibrous glass	0.10 to 0.11
Heavy gas-filled foam	0.12 to 0.16
Air-filled fibrous glass	0.22 to 0.23

Table 11 .... Thermal Conductivity of Some Insulating Materials[100]

Insulation	Temp F	k, Btu/(hr) (sq ft) (F deg per in.)
Urethane Foam, CCl_3F Blown		
Cut samples, initial	75	0.110
Cut samples, fully aged in air	75	0.157
Urethane Foam, CO_2 Blown	75	0.24
Polystyrene Rigid Foam	40	0.25
Mineral Wool	86	0.27
Glass Fiber	75	0.29
Cork Board	86	0.30

TRANSIENT HEAT FLOW

Often it is necessary to know the heat transfer and temperature distribution under unsteady state (varying with time) conditions. Examples are: cold storage temperature variations on starting or stopping a refrigeration unit, daily periodic variation of external air temperature and sun-load affecting the heat load of a cold storage room or wall temperatures, the time required to freeze a given material under certain conditions in a storage room, quick freezing of objects by direct immersion in brines, the time required for heating or cooling of fluids to certain temperatures, etc.

The fundamental equation for unsteady state conduction in solids or fluids in which there is no substantial motion is:

$$\frac{\partial t}{\partial \tau} = \alpha\left(\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}\right) \tag{48}$$

where the thermal diffusivity α is the ratio, $k/\rho c_p$ and k is the thermal conductivity, ρ, the density, and c_p, the specific heat. If α is large (high conductivity, low density and specific heat or both), heat will diffuse faster.

In some cases, it is desired to predict the rate of change of temperature of a body whose temperature can be considered to be uniform, e.g., a well-stirred reservoir of fluid, but whose temperature is changing because of either a net rate of heat gain or loss. In this case, the applicable equation may be written:

$$q_{net} = Mc_v\frac{dt}{d\tau} \tag{49}$$

where M is the mass of the body and c_v is the specific heat at constant volume of the body. For liquids and solids, the values of c_v and c_p are nearly equal and c_p may therefore be used with negligible error. The term, q_{net}, may include heat transfer by conduction, convection, or radiation and is the difference between the rate of heat transfer to the body and the heat transfer away from the body. Equation 49 is simply a statement of the first law of thermodynamics, neglecting work and mass flow. For systems in which work and mass flow must be considered, Equation 49 can be extended by adding the appropriate terms to represent these energy inputs. Chapter 1 can be used as a guide for writing these terms.

Problems involving transient convective heating can often be solved with reasonable accuracy by using steady-state relationships; however, it is well to remember that most of the relationships for convection are based on the assumption of steady temperature and flow. These relationships may not be valid for rapid transients. Although there is some information in the literature on convection with transient temperature, there is very little reported for the case of transient flow.

With the help of Equation 48, it is possible to derive expressions for temperature and heat flow variations at different instants and different locations. Most of the common cases have been solved and presented in graphical forms,[9,14,102] thus eliminating the need for solving original equations which become involved. In other cases, it is simpler to use Schmidt's graphical methods[15] or Dusinberre's numerical methods.[9]

The case of *freezing* an object in a cold-storage room consists of diffusing heat by conduction from the interior of the object and then transferring heat through the gas film resistance (which may be under natural air convection or forced air convection conditions). Quick freezing by immersion in brines is similar, except that the surface resistance is due to the liquid film. For these cases, Groeber's charts[14] and Heisler's charts[14] are available for objects resembling a slab, cylinder or a sphere. If the object cannot be approximated by these shapes, Newman[95] presents a method for considering odd shapes or cases where a part of the body is insulated. The rate of cooling or heating a cold storage room with large thermal inertia can also be calculated using these charts.

Regarding *other applications*, for unsteady state heating or cooling of reservoirs of fluids with the help of an immersion type refrigeration coil or by an external heat exchanger, Kern[95] gives the methods developed by Bowman, Fisher and Sanders. The effect of periodic temperature changes has been given in detail by Jakob.[14] Mackey[103] and Ghai[104] present the transient heat transfer phenomenon for thick walls under periodic outside air temperature variations. Leopold[105] deals with heat storage in cooling panels.

LETTER SYMBOLS USED IN CHAPTER 4

- A = area for heat transfer, square feet.
- A_{ef} = total effective area (Equation 7a, Table 8), square feet. $A_{ef} = A_s\phi + A_p$.
- A_m = mean area for heat transfer, square feet.
- A_p = area of prime surface, square feet.
- A_s = area of secondary surface, square feet.
- a_f = area of one side of one fin, square feet.
- B_{ij} = absorption factor or fraction of radiant energy emitted by surface i which is absorbed by surface j.
- b = breadth of a condensing surface, feet. For a vertical tube, $b = \pi d$; for a horizontal tube, $b = 2L$.
- C = fluid capacity rate, Btu per (hour) (Fahrenheit degree).
- C_c = capacity rate for cold fluid.
- C_h = capacity rate for hot fluid.
- C_{max} = larger capacity rate.
- C_{min} = smaller capacity rate.
- C_1 = first Boltzmann constant = 3.7413×10^{-5} (erg) (square centimeters) per second.
- C_2 = second Boltzmann constant = 1.4388 (centimeter) (Kelvin degrees).
- c = constant.
- c_p = specific heat at constant pressure, Btu per (pound mass) (Fahrenheit degree).
- c_v = specific heat at constant volume, Btu per (pound mass) (Fahrenheit degree).
- D_{b_1} = initial bubble diameter, feet.
- D_{b_2} = maximum bubble diameter, feet.
- D_e = hydraulic diameter, feet.
- D_{eq} = equivalent diameter, feet. (Equation 7a, Table 8.)
- D_o = outside diameter of a circular fin, feet.
- D'_o = outside diameter of a circular fin, inches.
- d = diameter of a tube, feet.
- d' = diameter of a tube, inches.
- F_1 = condensing coefficient factor (see Table 9).
- F_2 = condensing coefficient factor (see Table 9).
- F_{ij} = angle factor or fraction of radiant energy emitted by surface i which falls directly upon surface j.