must be used in the U-bend to take up the deformation, Δ. When welding fittings are used, the radii should be six times the outside diameter of the pipe. One other restriction is that the pipe should not be heavier than extra strong.

Manufacturers of pipe, or pipe fittings, often publish excellent treatises on the simplification of expansion problems. It is suggested that the engineer obtain such publications and also a text or handbook[6] on the subject before attempting to solve a problem on flexibility.

HANGERS AND SUPPORTS

In the preceding section, Equation 3 was developed to give the force of total constraint. This equation is appropriate regardless of the sign of Δt, positive or negative. It did not, however, allow for any bending moment but was for straight compression or tension.

TABLE 21. EQUATIONS FOR BENDING STRESS AND DEFLECTION BETWEEN SUPPORTS

TYPE OF SUPPORT	BENDING STRESS, PSI S	DEFLECTION, INCHES Y
Single span (free ends)	$S = \frac{0.75WL^2D}{I}$	$Y = \frac{22.5WL^4}{EI}$
Continuous Line	$S = \frac{0.5WL^2D}{I}$	$Y = \frac{4.5WL^4}{EI}$

where

W = Total weight (pipe, fluid, etc) pounds per foot.
D = O. D. of pipe, inches.
L = Length of span, feet.
E = Modulus of elasticity, pounds per square inch.
I = Moment of inertia, (inches)^4.
$I = \frac{\pi}{64}(OD^4 - ID^4)$, O. D. and I. D. in inches.

If the compressive force is great enough, or if the line is long enough, a slight eccentricity of the load application will cause a *buckling* of the line. This buckling is identical with the action of a column under excessive compression. In fact, a pipe line in compression is identical to a column. Column criteria can therefore be used for selecting the action of supports to resist lateral thrust.

A common equation for determining safe column loads is Euler's formula:

$$S = \pi^2 E \left(\frac{k}{l}\right)^2 \quad \text{(for cast iron see Equation 8)} \tag{5}$$

where

S = allowable column load, pounds per square inch.
k = radius of gyration, inches.
l = column length, inches.

$$k^2 = \frac{I}{A} = \frac{OD^2 + ID^2}{16}$$

Equation 5 is sometimes modified by a correction factor C to the following:

$$S = C\pi^2 E \left(\frac{k}{l}\right) \tag{6}$$

where

C = correction factor depending on end loading in accordance with the following table:

END CONDITIONS OF COLUMN		VALUE OF C
ONE END	OTHER END	
Fixed	Fixed	4.00
Fixed	Free	0.25
Fixed	Pin	2.25
Pin	Pin	1.00

From Equation 6,

$$l = \pi k \sqrt{\frac{CE}{S}} \tag{7}$$

This equation is not used for column construction but is sufficiently accurate for the work of a heating engineer.

The equation is modified for cast iron because of its poor tensile strength. It is generally acceptable to use one-half the value of S from Equation 5 for cast iron, so it becomes

$$S^1 = \frac{\pi^2 E}{2}\left(\frac{k}{l}\right)^2 \quad \text{for cast iron.} \tag{8}$$

It is permissible to allow the l/k ratio for steel to go as high as 120, but for cast iron the desirable maximum is taken as 70. It is not to be understood that a fracture will occur at l/k ratios in excess of those given, rather it is to be understood that lateral support should be given for those ratios and for those magnitudes of stress. If an expansion joint is used, this will, of course, relieve the compressive stress.

Example: For the problem given in the above section, the constraining force for a 1 in. pipe was 9630 lb. What is a required spacing for lateral support for this pipe?

Answer: Using Euler's formula, Equation 5 gives

$$S = \pi^2 E \left(\frac{k}{l}\right)^2$$

so

$$l = \pi k \sqrt{E/S}$$

For a 1 in. Schedule 40 steel pipe

k = 0.4205 inches.
E = 30,000,000 pounds per square inch.
S = 9630 ÷ 0.494 = 19500 psi
l = 51.8 inches.

Therefore, lateral supports on 4 foot centers would be adequate.

The other type of load requiring some support is the bending stress due to the weight of the pipe, its insulation, and of the fluid being carried.