

Fig. 23 . . . . Bootstrap Air-Cycle Refrigeration System for *Example 2*

Bleed Air Circuit

From Fig. 16, read $E_1 = 0.86$. From Fig. 17, read $E_2 = 0.86$.

$$E_1 = \frac{T_1 - T_2}{T_1 - T_4} = \frac{\Delta T_{1-2}}{T_1 - T_4}$$

$$\Delta T_{1-2} = E_1(T_1 - T_4) = 0.86(685 - 567) = 101 \text{ F deg}$$

$$T_2 = T_1 - \Delta T_{1-2} = 685 - 101 = 584 \text{ R}$$

$$(T_{AVG})_{1-2} = \frac{T_1 + T_2}{2} = \frac{685 + 584}{2} = 634.5 \text{ R}$$

From Fig. 18, read $\sigma\Delta P_{1-2} = 0.193$ in. Hg

$$\Delta P_{1-2} = \frac{(\sigma\Delta P_{1-2})(T_{AVG})_{1-2}}{17.35\left(P_1 - \frac{\Delta P_{1-2}}{2}\right)}$$

$$= \frac{(0.193)(634.5)}{17.35\left(53 - \frac{\Delta P_{1-2}}{2}\right)} = 0.137 \text{ in. Hg}$$

(Note that this equation is arranged in a form convenient for solution on a slide rule, but requiring a trial-and-error process. It can be rearranged in quadratic form and solved directly for ΔP_{1-2}.)

$$P_2 = P_1 - \Delta P_{1-2} = 52 - 0.14 = 51.86 \text{ in. Hg abs.}$$

The compressor flow factor is

$$F_{CF} = \frac{W_B\sqrt{T_2}}{D^2P_2}$$

Since the compressor tip diameter D is 6.2 in. (Fig. 19),

$$F_{CF} = \frac{70\sqrt{584}}{(6.2)^2(51.86)} = 0.85$$

From Fig. 19, read the compressor Mach number $M = 0.85$.

$$M = \frac{DN}{11{,}235\sqrt{T_2}}$$

where

$$11{,}235 = \frac{12 \times 60\sqrt{\gamma gR}}{\pi} \text{ (inches) (rpm) per } \sqrt{\text{deg Rankine}}.$$

R = gas constant for air = 53.3 ft lb per (lb) (Rankine deg).

$\pi = 3.1416$

The compressor speed is

$$N = \frac{11{,}235\sqrt{T_2}(M)}{D} = \frac{11{,}235\sqrt{584}\ (0.845)}{6.2} = 37{,}300 \text{ rpm}$$

From Fig. 19, read the compressor efficiency $\eta_c = 77$ percent. The compression factor is

$$Y_c = (r_c)^{(\gamma-1)/\gamma} - 1 = (P_2/P_1)^{(\gamma-1)/\gamma} - 1$$

$$= (1.62)^{0.286/1.286} - 1 = 0.1463$$

$$\Delta T_{2-3} = \frac{Y_cT_2}{\eta_c} = \frac{(0.1463)(584)}{0.77} = 111 \text{ F deg}$$

$$T_3 = T_2 + \Delta T_{2-3} = 584 + 111 = 695 \text{ R}$$

$$P_3 = P_2\left(\frac{P_3}{P_2}\right) = 51.86(1.62) = 84 \text{ in. Hg abs}$$

$$E_2 = \frac{T_3 - T_4}{T_3 - T_5} = \frac{\Delta T_{3-4}}{T_3 - T_5}$$

$$\Delta T_{3-4} = E_2(T_3 - T_5) = 0.86(695 - 567) = 110 \text{ F deg}$$

$$T_4 = T_3 - \Delta T_{3-4} = 695 - 110 = 585 \text{ R}$$

$$(T_{AVG})_{3-4} = \frac{T_3 + T_4}{2} = \frac{695 + 585}{2} = 640 \text{ R}$$

From Fig. 20, read $\sigma\Delta P_{3-4} = 3.22$ in. Hg.

$$\Delta P_{3-4} = \frac{(\sigma\Delta P_{3-4})(T_{AVG})_{3-4}}{17.35\left(P_3 - \frac{\Delta P_{3-4}}{2}\right)} = \frac{(3.22)(640)}{17.35\left(84 - \frac{\Delta P_{3-4}}{2}\right)}$$

$$= 1.43 \text{ in. Hg.}$$

$$P_4 = P_3 - \Delta P_{3-4} = 84 - 1.43 = 82.57 \text{ in. Hg abs.}$$

The turbine pressure ratio r_t is $P_4/P_5 = 82.57/32 = 2.58$. The turbine Mach number is

$$M = \frac{N}{1725\sqrt{T_4}} = \frac{37{,}300}{1725\sqrt{585}} = 0.894$$

From Fig. 21, read the turbine flow factor $F_{TF} = 13.7$.

$$F_{TF} = \frac{W_B\sqrt{T_4}}{P_4A}$$

$$W_B = \frac{P_4A}{\sqrt{T_4}}(F_{TF}) = \frac{(82.57)(1.497)}{\sqrt{585}}(13.7)$$

$$= 70 \text{ lb per min.}$$

This checks with the assumed value of W_B. The expansion factor Y_t is

$$Y_t = 1 - \frac{1}{(r_t)^{(\gamma-1)/\gamma}} = 1 - \frac{1}{(2.58)^{0.286/1.286}} = 0.235$$

The isentropic temperature drop between points 4 and 5 is

$$(\Delta T_t)_{4-5} = T_4Y_t = (585)(0.235) = 137.5 \text{ F deg.}$$

The turbine velocity factor is

$$F_V = \frac{N}{2750\sqrt{\Delta T_{t_{4-5}}}}$$

where

$$2750 = \sqrt{Jg_c\eta\left(\frac{12 \times 60}{\pi \times 6.5}\right)^2} \text{ rpm per } \sqrt{\text{degrees Rankine}}.$$

$$F_V = \frac{37{,}300}{2750\sqrt{137.5}} = 1.157$$

From Fig. 22, read the turbine efficiency $\eta_t = 83.3$ percent.

$$\Delta T_{t_{4-5}} = \eta_t(\Delta T_t)_{4-5} = 0.833(137.5) = 114.5 \text{ F deg}$$

$$\text{Turbine Power} = \frac{\text{Compressor Power}}{\eta_m}$$

$$W_Bc_p\Delta T_{t_{4-5}} = \frac{W_Bc_p\Delta T_{2-3}}{\eta_m}$$

$$\Delta T_{t_{4-5}} = \frac{\Delta T_{2-3}}{\eta_m} = \frac{111}{0.97} = 114.5 \text{ F deg.}$$

This value of $\Delta T_{t_{4-5}}$ checks with the value calculated for the equation $\Delta T_{t_{4-5}} = \eta_t(\Delta T_t)_{4-5}$ and indicates that a balance exists between turbine power and compressor power. This also shows that the assumed value of compressor pressure ratio r_c is correct.

$$T_5 = T_4 - \Delta T_{t_{4-5}} = 585 - 114.5 = 470.5 \text{ R } (10.5 \text{ F}).$$

Primary Heat Exchanger Cooling Air Circuit

$$\Delta P_{6-7} = 30.65 - 30.21 = 0.44 \text{ in. Hg}$$

$$(P_{AVG})_{6-7} = \frac{P_6 + P_7}{2} = \frac{30.65 + 30.21}{2} = 30.43 \text{ in. Hg abs}$$

$$\Delta T_{6-7} = \frac{W_B}{W_{C1}}(\Delta T_{1-2}) = \frac{70}{200}(101) = 35 \text{ F deg}$$

$$T_7 = T_6 + \Delta T_{6-7} = 567 + 35 = 602 \text{ R}$$

$$(T_{AVG})_{6-7} = \frac{T_6 + T_7}{2} = \frac{567 + 602}{2} = 584.5 \text{ R}$$

$$\sigma\Delta P_{6-7} = 17.35\frac{(P_{AVG})_{6-7}}{(T_{AVG})_{6-7}}(\Delta P_{6-7})$$

$$= 17.35\left(\frac{30.43}{584.5}\right)(0.44) = 0.397 \text{ in. Hg}$$

From Fig. 18, read $W_{C1} = 200$ lb per min. This value checks with the assumed value for W_{C1}.

Secondary Heat Exchanger Cooling Air Circuit

$$\Delta P_{8-9} = 30.65 - 30.21 = 0.44 \text{ in. Hg}$$

$$(P_{AVG})_{8-9} = \frac{P_8 + P_9}{2} = 30.43 \text{ in. Hg abs}$$

$$\Delta T_{8-9} = \frac{W_B}{W_{C2}}(\Delta T_{3-4}) = \frac{70}{138}(110) = 56 \text{ F deg}$$

$$T_9 = T_8 + \Delta T_{8-9} = 567 + 56 = 623 \text{ R}$$

$$(T_{AVG})_{8-9} = \frac{T_8 + T_9}{2} = 595 \text{ R}$$

$$\sigma\Delta P_{8-9} = 17.35\frac{(P_{AVG})_{8-9}}{(T_{AVG})_{8-9}}(\Delta P_{8-9})$$

$$= 17.35\left(\frac{30.43}{595}\right)(0.44) = 0.391 \text{ in. Hg}$$

From Fig. 20, read $W_{C2} = 138$ lb per min. This value checks with the originally assumed value for W_{C2}.

DESIGN OF SYSTEMS FOR OTHER AIR FLOW CAPACITY

The test performance curves for the fan, compressor, turbines and heat exchangers in fixed-size systems may be used to design systems with larger or smaller air-flow capacity. The methods used will be discussed for each of the components.

Fan

Fig. 14 presents the performance of a 3.5-in. tip diameter axial-flow fan. The performance of any geometrically similar fan with a tip diameter, D, between 3 and 10 in. may be predicted, with reasonable accuracy, by using Fig. 14 and the following equations.

$$\left(\frac{W_C\sqrt{T_2}}{P_2}\right)_{2.5} = \left(\frac{W_C\sqrt{T_2}}{P_2}\right)_D\left(\frac{3.5}{D}\right)^3$$

$$\left(\frac{W_B\Delta T_{2-3}}{P_2\sqrt{T_2}}\right)_{2.5} = \left(\frac{W_B\Delta T_{2-3}}{P_2\sqrt{T_2}}\right)_D\left(\frac{3.5}{D}\right)^3$$

$$\left(\frac{P_2}{P_1}\right)_{2.5} = \left(\frac{P_2}{P_1}\right)_D$$

Compressor

Fig. 19 presents the performance of a 6.2 in. tip diameter centrifugal compressor with backward-curved impeller vanes. The curve is actually presented in dimensionless form so that it is directly applicable, with reasonable accuracy, to any geometrically similar compressor with an impeller tip diameter, D, between approximately 4 and 12 in.

$$(F_{CF})_{2.5} = (F_{CF})_D \quad \text{or}$$

$$\left(\frac{W_B\sqrt{T_2}}{D^2P_2}\right)_{6.2} = \left(\frac{W_B\sqrt{T_2}}{D^2P_2}\right)_D$$

$$(M)_{6.2} = (M)_D$$

or

$$\left(\frac{DN}{11235\sqrt{T_2}}\right)_{6.2} = \left(\frac{DN}{11235\sqrt{T_2}}\right)_D$$

$$\left(\frac{P_2}{P_1}\right)_{6.2} = \left(\frac{P_2}{P_1}\right)_D$$

$$(\eta_c)_{6.2} = (\eta_c)_D$$

Turbine

Figs. 12 and 13 present performance of a straight radial inward flow turbine wheel with a full admission nozzle. The wheel tip diameter is 3.5 in. These curves may be used to predict performance, with reasonable accuracy, for any geometrically similar turbine wheel with a tip diameter, D, between 3 and 10 in.

$$(F_V)_{3.5} = (F_V)_D\left(\frac{D}{3.5}\right)$$

or

$$\left(\frac{N}{5111\sqrt{(\Delta T_t)_{3-4}}}\right)_{3.5} = \left(\frac{N}{5111\sqrt{(\Delta T_t)_{3-4}}}\right)_D\left(\frac{D}{3.5}\right)$$

$$\left(\frac{W_B\sqrt{T_3}}{15.63P_4A_n}\right)_{3.5} = \left(\frac{W_B\sqrt{T_3}}{15.63P_4A_n}\right)_D$$

$$\left(\frac{P_3}{P_4}\right)_{3.5} = \left(\frac{P_3}{P_4}\right)_D$$

$$\left(\frac{N}{\sqrt{T_3}}\right)_{3.5} = \left(\frac{N}{\sqrt{T_3}}\right)_D\left(\frac{D}{3.5}\right)$$

$$(A_n)_D = (A_n)_{3.5}\left(\frac{D}{3.5}\right)^2$$

Figs. 21 and 22 present performance of an inward flow, radial turbine wheel with forward-curved vanes and a full admission nozzle. The wheel tip diameter is 6.5 in. These two curves may be used, with reasonable accuracy, to predict the performance of any geometrically similar turbine wheel with