

The connection of the velocity of sound with the flow of fluids has already been noted. Its most important application is to the flow of gases through a converging tube or nozzle. If it is assumed that the inlet velocity of the fluid,  $V_1$ , is negligible, Equation 24 will reduce to

$$V_2 = \sqrt{\left(\frac{2gk}{k-1}\right) \frac{p_1}{\rho_1} \left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}\right]} \quad (33)$$

Let  $W$  represent the weight of gas flowing through the converging tube in a unit of time, and  $A_2$  the area at the throat; then,

$$W = A_2 V_2 / v_2 \text{ or } V_2 = W v_2 / A_2 \quad (34)$$

Substituting this, as well as the relation  $p_1 v_1^k = p_2 v_2^k$ , in Equation 33 gives,

$$W = A_2 \sqrt{\left(\frac{2gk}{k-1}\right) \frac{p_1}{v_1} \left[\left(\frac{p_2}{p_1}\right)^{\frac{2}{k}} - \left(\frac{p_2}{p_1}\right)^{\frac{k+1}{k}}\right]} \quad (35)$$

If this is computed and the figures are plotted, the curved line (partly solid and partly broken) of Fig. 6 is found. The maximum value of  $\frac{p_2}{p_1}$  may be computed by differentiating  $W$  with respect to  $p_2$  and equating the result to zero. This operation produces the formula:

$$\frac{p_2}{p_1} = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}} \quad (36)$$

For air, with  $k = 1.40$ ,  $\frac{p_2}{p_1} = 0.53$ .

#### Critical Pressure and Critical Flow

Actually, the broken part of the curve is not attained for the flow in the nozzle. If the ratio of  $p_2$  to  $p_1$  is decreased from unity, the weight

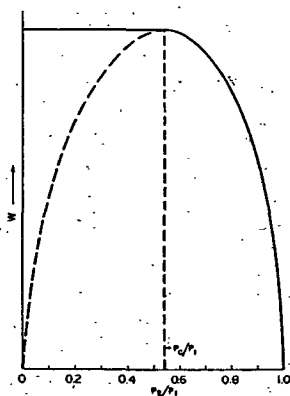


FIG. 6. RELATION OF FLOW OF GAS TO PRESSURE DROP IN A CONVERGING TUBE.

rate of discharge, as well as the volume, increases from zero to a maximum, as shown by the solid section of the curve in Fig. 6; thereafter, as  $p_2/p_1$  is decreased further, the discharge is constant, as indicated by the horizontal line. The value of  $p_2$  at the maximum point is called the critical pressure, or  $p_c$ , and from Equation 27 it is seen that  $p_c$  is approximately 53 per cent of  $p_1$  when air is flowing.

To find the velocity at the critical pressure, let us assume that the upstream velocity  $V_1$  in Equation 22 is so small as to be negligible. Using the subscript  $c$  to indicate conditions at the critical point, we have

$$\frac{V_c^2}{2g} = \left(\frac{k}{k-1}\right) (p_1 v_1 - p_c v_c) \text{ or,} \quad (37)$$

$$V_c = \sqrt{\left(\frac{2gk}{k-1}\right) (p_1 v_1 - p_c v_c)}$$

Substituting Equations 23 and 36, and rearranging, Equation 37 becomes

$$V_c = \sqrt{\left(\frac{2gk}{k+1}\right) p_1 v_1} \quad (38)$$

and

$$V_c = \sqrt{\frac{gk p_c}{\rho_c}} \quad (39)$$

Comparing Equation 39 with Equation 25, it will be seen that the velocity at the throat is equal to the velocity of sound at the critical pressure.

Critical flow is attained only in converging tubes, in nozzles, and in orifices with a well-rounded approach. It does not occur in sharp-edged orifices or in nozzles having an expanding outlet section. The so-called critical flow prover uses this property of constant rate of flow above the critical pressure, and finds application as a flow regulator and a quantity-rate meter; in either case, the theoretical rate of flow may be computed from Equation 38, multiplying  $V_c$  by the area of the constriction to obtain the volume rate of flow.

In developing the working equations for orifices and nozzles, it is customary to start with

$$V_2^2 - V_1^2 = 2gh_f \quad (40)$$

This may be derived from the Bernoulli equation or from the relations of falling bodies. Now, since

$$A_1 V_1 = A_2 V_2 = Q_s \quad (41)$$

in which  $Q_s$  is the discharge rate in cubic feet per second,

$$\frac{Q_s^2}{A_1^2} - \frac{Q_s^2}{A_2^2} = 2gh_f \quad (42)$$

Transposing,

$$Q_s = \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2gh_f} \quad (43)$$

or