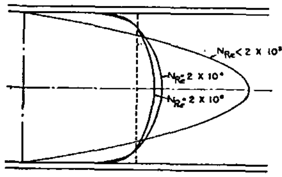

Fig. 5 .... Comparison of Velocity Profiles for 3 Different Reynolds Numbers but for Same Average Velocity

the pressure loss is less than 10 percent of the initial pressure. When the loss in head is high, the formula to be used for gases is

$$\frac{p_1^2 - p_2^2}{p_1^2} = \frac{flV_1^2}{gd\,p_1v_1} \tag{13}$$

which may be arranged to give the loss in pressure,

$$p_1 - p_2 = p_1\left[1 - \sqrt{1 - \frac{flV_1^2}{gd\,p_1v_1}}\right] \tag{14}$$

Pressure Loss in Non-Circular Pipes

The formulas for friction loss in pipes are based on the use of pipes of circular cross-section. The same formulas may be extended to noncircular sections, by suitable modification. In the basic formula, Equation 8, the internal diameter d is to be replaced by the hydraulic diameter d_H defined by the equation:

$$d_H = \frac{4 \times \text{area of cross-section}}{\text{wetted perimeter of cross-section}} \tag{15}$$

For example, in a rectangular duct, 1 ft by 2 ft, the cross-section area is 2 sq ft, and the perimeter 6 ft. Then the hydraulic diameter will be $d_H = (4 \times 2)/6 = 1\frac{1}{3}$ ft.

In the case of a round pipe,

$$d_H = \frac{4 \times \pi d^2/4}{\pi d} = d \tag{16}$$

In computing the Reynolds number, and from that the friction factor, the hydraulic diameter is not to be used. A better approximate procedure is to replace the length in the Reynolds number by the shortest dimension plus one-fourth of the hydraulic diameter. Thus, in a duct of dimension a x b where a < b, N_{Re}, for the purposes of calculating friction factors, is

$$N_{Re} = (a + 0.25d_H)V\rho/\mu \tag{17}$$

This value of N_{Re} may be used in Equation 10 for laminar flow, and in Equation 12 or Fig. 4 for turbulent flow. The error in the approximation is somewhat greater for laminar than for turbulent flow. In the former case, the relative error may be as much as 10 percent, while in the latter it almost always is less than 3 percent.

FLOW OF COMPRESSIBLE FLUIDS

In the flow of compressible fluids, the large density variations make it impracticable to use the Bernoulli equation, (Equation 7). In certain special cases, however, the exact equations for compressible flow may be stated. If flow occurs with no friction or other internal irreversibility Equation 6 becomes

$$\frac{1}{2g}\,dV^2 + \frac{dp}{\rho} = 0 \tag{18}$$

If, in addition, the flow is adiabatic,

$$p\rho^{-k} = p_1\rho_1^{-k} \tag{19}$$

so that Equation 18 becomes

$$\frac{1}{2g}\,dV^2 + \frac{p_1^{1/k}}{\rho_1}\frac{dp}{p^{1/k}} = 0 \tag{20}$$

or by integration,

$$\frac{1}{2g}(V_2^2 - V_1^2) + \frac{k}{k-1}\frac{p_1}{\rho_1}\left[\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} - 1\right] = 0 \tag{21}$$

This extension to compressible flow of Bernoulli's equation reduces to the more familiar form if the pressure change is small.

The ratio of specific heats, k, is used extensively in fluid dynamics; values of k for various gases are given in Table 2.

It is convenient in the analysis of compressible flow to introduce the velocity of propagation of pressure impulses or, more familiarly, the sonic velocity, a. For perfect gases this is given by the equation:

$$a^2 = kgp/\rho = kgRT \tag{22}$$

Accordingly, Equation 21 may be written

$$\tfrac{1}{2}(V_2^2 - V_1^2) + \frac{a_1^2}{k-1}\left[\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} - 1\right] = 0 \tag{23}$$

or, by rearrangement,

$$\frac{p^2}{p_1} = \left[1 - \frac{k-1}{a_1}\left(\frac{V_2^2 - V_1^2}{2}\right)\right]^{\frac{k}{k-1}} \tag{24}$$

which permits the calculation of the ratio of pressures at entrance and exit of the steady flow device—pipe, orifice, or nozzle. From Equations 19 and 22 it follows that

$$\frac{a_2^2}{a_1^2} = \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} \tag{25}$$

so that

$$\frac{k-1}{2}(V_2^2 - V_1^2) + a_2^2 - a_1^2 = 0 \tag{26}$$

and

$$\frac{a_2^2}{a_1^2} = \frac{1 + \frac{k-1}{2}\frac{V_1^2}{a_1^2}}{1 + \frac{k-1}{2}\frac{V_2^2}{a_2^2}} \tag{27}$$

The ratio of flow velocity to sonic velocity is known as the Mach number,

$$M = V/a$$

This parameter is particularly useful in compressible flow analysis. In general, if $M \leq 0.1$ the flow may be considered

to be incompressible. This is generally true in heating and ventilating air ducts.

In terms of the Mach number

$$\frac{a_2^2}{a_1^2} = \frac{T_2}{T_1} = \frac{1 + \frac{k-1}{2}M_1^2}{1 + \frac{k-1}{2}M_2^2} \tag{28}$$

and

$$\frac{p_2}{p_1} = \left(\frac{1 + \frac{k-1}{2}M_1^2}{1 + \frac{k-1}{2}M_2^2}\right)^{\frac{k}{k-1}} \tag{29}$$

The quantity,

$$p^s = p\left(1 + \frac{k-1}{2}M_1^2\right)^{\frac{k}{k-1}} \tag{30}$$

is called the stagnation pressure, and gives a measure of pressure energy. For incompressible flow

$$p^s = p + \frac{1}{2g}\rho V^2 = p + q \tag{31}$$

where

$$q = \frac{1}{2g}\rho V^2 \tag{32}$$

and is the dynamic pressure. A total head tube measures stagnation pressure directly.

From Equation 29 it follows that for frictionless, adiabatic flow

$$p_2^s = p_1^s \tag{33}$$

This represents another extension of the Bernoulli equation to compressible flow. Friction will cause a loss in pressure energy.

Ideal Flow through Nozzle or Orifice

The majority of low-head measuring systems depend upon a correlation between pressure drop, area, and quantity of flow. The basic formulas may be stated on the assumption that the flow is frictionless and adiabatic. Designating the main stream by station 1, and flow at some measuring restriction by station 2, the flow in pounds per second is

$$w = \rho_1A_1V_1 = \rho_2A_2V_2 \tag{34}$$

or, in terms of Mach number,

$$w = A_2M_2\sqrt{kgp_2\rho_2} \tag{35}$$

According to Equation 29

$$\left(\frac{p_1}{p_2}\right)^{\frac{k-1}{k}} = \frac{1 + \frac{k-1}{2}M_2^2}{1 + \frac{k-1}{2}M_1^2} \tag{36}$$

from which

$$M_2^2 = \frac{2}{k-1}\left(\frac{1 + \frac{k-1}{2}M_1^2}{1 - M_1^2/M_2^2}\right)\left[\left(\frac{p_1}{p_2}\right)^{\frac{k-1}{k}} - 1\right] \tag{37}$$

Table 2 .... Ratio of Specific Heat at Constant Pressure to Specific Heat at Constant Volume for Compressible Fluids

Compressible Fluid	Ratio $k = c_p/c_v$
Helium and other monatomic gases..........	1.66
Air and other diatomic gases..............	1.40
Ammonia and hydrogen sulfide..............	1.34
Carbon dioxide, methane, natural gas, superheated steam, moist steam down to a quality of 97 percent..............................	1.28 to 1.32
Sulfur dioxide, ethylene, acetylene........	1.24 to 1.26

so that

$$w = A_2\sqrt{\frac{1 + \frac{k-1}{2}M_1^2}{1 - M_1^2/M_2^2}}\sqrt{\frac{2kgp_2\rho_2}{k-1}\left[\left(\frac{p_1}{p_2}\right)^{\frac{k-1}{k}} - 1\right]} \tag{38}$$

If the initial velocity is sufficiently small, M_1^2 will be negligible so that

$$w = A_2\sqrt{\frac{2kgp_2\rho_2}{k-1}\left[\left(\frac{p_1}{p_2}\right)^{\frac{k-1}{k}} - 1\right]}$$

$$= A_2\sqrt{\frac{2kgp_1\rho_1}{k-1}\left(\frac{p_2}{p_1}\right)^{\frac{k+1}{k}}\left[\left(\frac{p_1}{p_2}\right)^{\frac{k-1}{k}} - 1\right]} \tag{39}$$

If this is computed and the figures are plotted, the curved line (partly solid and partly broken) of Fig. 6 is found. The maximum value of $\frac{p_2}{p_1}$ may be computed by differentiating w with respect to p_2 and equating the result to zero. This operation produces the equation:

$$\frac{p_2}{p_1} = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}} \tag{40}$$

For air, with $k = 1.40$, $\frac{p_2}{p_1} = 0.53$.

Actually, the broken part of the curve is not attained for

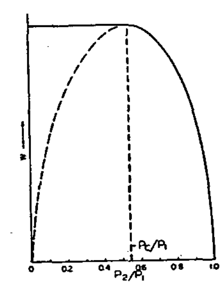

Fig. 6 .... Relation of Flow of Gas to Pressure Drop in a Converging Tube