in the case of some slow-drying materials, such as soap and wood, then no constant rate will appear, and the whole of the drying process will be in the falling-rate period. This period, in the most general case, can be divided into two zones which may be termed (1) the zone of unsaturated surface drying, and (2) the zone where internal liquid flow controls.

The zone of unsaturated surface drying follows immediately after the critical point and results from a progressively decreasing wetted surface. With the surface no longer completely wetted, dry portions of the solid protrude into the air film, so that the rate of evaporation per unit of total surface is reduced. The effective wetted surface in this zone is frequently a linear function of the water content, so that the curve representing rate of drying *vs.* water content of the solid is straight in this region, as shown by line AD in Fig. 3. The mechanism of drying is essentially the same as during the constant-rate period.

The zone where internal liquid flow is in control, is usually the second zone of the falling-rate period. In this phase the rate of internal liquid movement controls the drying rate, and in drying to low moisture contents, this period may be the principal factor determining the drying time.

Studies of internal moisture flow have indicated the possibility of several controlling mechanisms, the more significant ones having been postulated previously as diffusion, capillarity and pressure gradients due to shrinkage. Of these mechanisms, internal moisture movement by diffusion has been treated extensively, while capillary flow and flow caused by shrinkage and pressure gradients, have received only preliminary consideration.

When diffusion does control in the falling-rate period, it obeys the same fundamental laws of diffusion as those applying to the diffusion of heat. On this basis, the integrated diffusion equation for the falling-rate period (for the case where the surface is dry or at its equilibrium moisture content and the solid has a uniform initial moisture distribution) expresses the average moisture content as a function of time as follows:

$$\frac{W - W_e}{W_c - W_e} = \frac{8}{\pi^2}\left[e^{-\theta d(\pi/2L)^2} + \frac{1}{9}e^{-9\theta d(\pi/2L)^2} + \frac{1}{25}e^{-25\theta d(\pi/2L)^2} + \cdots\right] \quad (12)$$

where

W, W_c, W_e = the moisture contents, on a dry basis, at any time θ; at $\theta = 0$, the start of the diffusional flow period; and in equilibrium with the external conditions, respectively, pounds of water per pound of dry solid.

d = the liquid diffusivity, square feet per hour.

L = one-half the thickness of the solid layer through which the liquid is diffusing, feet.

In Equation 12 it is assumed that evaporation is occurring from two opposite faces of the solid. When evaporation occurs from only one surface, substitute the total thickness of the solid layer for L in Equation 12.

Equation 12 is based on the assumption that d is constant. However, this is rarely true, and d has been shown to vary with moisture content, temperature and humidity.[7]

When the time becomes large, a limiting form of Equation 12 is obtained as follows:

$$\frac{W - W_e}{W_c - W_e} = \frac{8}{\pi^2}e^{-\theta d(\pi/2L)^2} \quad (13)$$

From Equation 13 an expression for the rate of drying may be derived to give

$$\frac{dW}{d\theta} = -\frac{\pi^2 d}{4L^2}(W - W_e) \quad (14)$$

where $dW/d\theta$ = drying rate, pounds per (hour) (pound dry material).

Equation 14 states that the rate of drying, when internal diffusion controls for long times, is directly proportional to the free moisture content $(W - W_e)$, the liquid diffusivity d, and that the time of drying varies as the square of the material thickness. However, Equation 14 holds only when $(W - W_e)/(W_c - W_e) < 0.6$. When this ratio exceeds 0.6, the curve of drying rate *vs.* moisture content is concave upward.

Equations 12, 13, and 14 hold only for a slab-shaped solid, the length of which is large compared with its thickness.

The falling rate frequently can be expressed with fair accuracy over the required range of moisture content by an equation similar to Equation 14:

$$\left(\frac{dW}{d\theta}\right)_f = -K(W - W_e) \quad (15)$$

where K is a function of the constant rate as follows:

$$K = \frac{(dW/d\theta)_c}{(W_c - W_e)} \quad (16)$$

where

$(dW/d\theta)_c$ = the constant drying rate, pounds per (hour) (pound dry material).

W_c = the critical moisture content, pounds per pound dry material.

Substituting in Equation 16 the proper expression for $(dW/d\theta_c)$ the value of K becomes

$$K = \frac{h_t(t_a - t_s)}{\rho_s L\lambda(W_c - W_e)} \quad (17)$$

and hence, the falling rate for this case is given by

$$\left(\frac{dW}{d\theta}\right)_f = -\frac{h_t(t_a - t_s)(W - W_e)}{\rho_s L\lambda(W_c - W_e)} \quad (18)$$

For materials obeying Equation 18, the drying time varies directly as the thickness. When the surface temperature in the constant-rate period is at the wet-bulb temperature, t_w can be substituted for t_s and $0.0128\,G^{0.8}$ can be substituted for h_t in Equations 17 and 18.

The drying time for each case of the falling-rate period may be obtained by integration of Equations 14 and 18, respectively, to give:

1. Diffusion law

$$\theta_f = \frac{4L^2}{d\pi^2}\log_e\left(\frac{W_c - W_e}{W - W_e}\right) \quad (19)$$

2. Proportional-to-thickness law

$$\theta_f = \frac{\rho_s L\lambda(W_c - W_e)}{h_t(t_a - t_s)}\log_e\left(\frac{W_c - W_e}{W - W_e}\right) \quad (20)$$