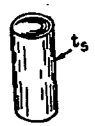
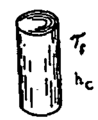
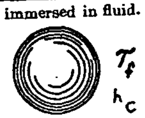
TABLE 7. ANALYTICAL SOLUTIONS FOR HEAT CONDUCTION IN VARIOUSLY SHAPED SOLIDS (*Concluded*)

SHAPE OF SOLID	BOUNDARY CONDITIONS	DATA AVAILABLE IN GRAPHS
Cylinder of infinite axial dimension 	The surface temperature is suddenly changed from the initial (uniform) temperature.	Temperature distribution as a function of time. Reference: (10) p. 265.
		Heat flow from surface as a function of time. Multiply temperature difference between surface and fluid by surface conductance.
	The surface temperature suddenly begins to increase linearly with time.	Temperature distribution as a function of time. Reference: (10) p. 269.
Cylinder of infinite axial dimension immersed in a fluid. 	The surrounding fluid suddenly changes from the initial (uniform) temperature of the cylinder.	Temperature distribution as a function of time. References: (4) p. 36; (5) pp. V-16, V-35, V-43, V-48; (10) pp. 278, 286; (15).
		Heat flow from the surface as a function of time. References: (5) p. V-16; (10) p. 278.
	The temperature of the surrounding fluid changes sinusoidally.	Temperature distribution as a function of time. Reference: (5) p. VI-34.
		Heat flow from the surface as a function of time. Reference: (5) p. VI-36.
Sphere 	The temperature of the surface is suddenly changed from the initial uniform temperature.	Temperature distribution as a function of time. References: (5) p. V-23; (10) pp. 264, 265.
	The temperature at the surface suddenly begins to change as a linear function of time.	Temperature distribution as a function of time. Reference: (10) p. 269.
Sphere immersed in fluid. 	The temperature of the surrounding fluid suddenly changes from the initial uniform sphere temperature.	Temperature distribution as a function of time. References: (4) p. 36; (5) pp. V-21, V-35, V-44; (10) pp. 281, 282; (4).
		Heat flow as a function of time. References: (5) p. V-21; (10) p. 281.
Rectangular bar of infinite length.	Any of the above noted boundary conditions for a slab.	Temperature distribution as a function of time. Combine solutions as indicated in Refs. 16 and 17.
Parallelopiped (rectangular)	Any of the above noted boundary conditions for a slab.	Temperature distribution as a function of time. Combine solutions as indicated in Refs. 16 and 17.
Cylinder of finite length.	Any of the boundary conditions given above for a cylinder and a slab.	Temperature distribution as a function of time. Combine solutions as indicated in Refs. 16 and 17.
Hollow cylinder of infinite exterior radius.	The temperature of the surface suddenly changes from the initial (uniform) temperature.	Temperature distribution as a function of time. Combine solutions as indicated in Refs. 16 and 17.
		Heat flow at the surface as a function of time. Reference: (10) p. 267.

Consider the slab to be divided, as shown in Fig. 9, by n equidistant planes parallel to the slab surface and a distance Δx apart. Let the temperature of the slab at any plane and any time (θ) be denoted by $T_{x,\theta}$. Then the temperature of the slab at the two adjacent planes at the same

time will be denoted as $T_{(x+\Delta x,\theta)}$ and $T_{(x-\Delta x,\theta)}$. In a similar manner the temperature of the x plane at a time $\Delta\theta$ later will be $T_{(x,\theta+\theta\Delta)}$.

In accordance with this nomenclature, the temperature at any plane x and time $\theta + \Delta\theta$ is given as

$$T_{(x,\theta+\Delta\theta)} = \frac{T_{(x+\Delta x,\theta)} + T_{(x-\Delta x,\theta)}}{2} \tag{18}$$

which may be interpreted as follows. The temperature of the slab at any plane, x, and any time, θ, is equal to the average temperature of the two adjacent planes obtained at the time $(\theta - \Delta\theta)$.

The time interval $\Delta\theta$ is determined by the equation

$$\Delta\theta = \frac{\Delta x^2}{2a} \tag{19}$$

Omitting the graphical construction at the slab boundaries, reference to Fig. 9 demonstrates the graphical method by means of which the temperature at each plane is determined at successive intervals of time in accordance with Equation 18.

For the problem stated, the boundary condition at the insulated surface is specified by the equation

$$\frac{\partial T}{\partial x} = 0$$

and at the uninsulated face by the equation

$$h(T_\infty - T) = -k\frac{\partial T}{\partial x}$$

In terms of finite differences these two equations (employing nomenclature established by Fig. 9) become

$$\frac{T_F - T_E}{\Delta x} = 0 \quad \text{or} \quad T_F = T_E \quad \text{at} \quad x = L$$

and

$$h(T_\infty - T_{A'}) = -k\frac{(T_B - T_A)}{\Delta x} \quad \text{at} \quad x = 0$$

or

$$\frac{T_\infty - T_{A'}}{k/h} = -\frac{T_A - T_B}{\Delta x}$$

The details of the graphical construction are best obtained by inspection of Fig. 9. Note that the line (0,0,0′) used to initiate the graphical construction, is the only one drawn to the slab boundary A'. The numbered points indicate temperatures at the sub-slab boundaries at 1,2,3, etc., time intervals $(\Delta\theta)$ after the slab is exposed to the high temperature.

For transient heat flow in two dimensions, and also, for steady state conduction, numerical methods of solution are available in the literature.[5,10,12,19] These numerical methods are applicable to three dimensional problems, although the calculations involved normally become too tedious for most applications of the method. An additional technique of solution for one and two dimensional problems in transient conduction results from