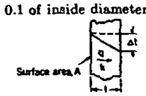
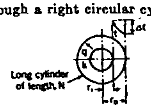
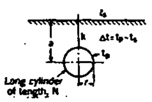
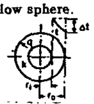
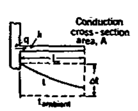
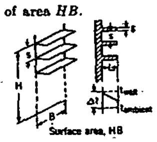
Table 6 . . . . Solutions for Some Steady-State Thermal Conduction Problems[a, b]

No.	System	Expressions for the Resistance R Entering into the Equation: $q = \Delta t/R$ (Btu per hour)
1.	Flat wall or curved wall if curvature is small (wall thickness less than 0.1 of inside diameter). 	$R = \dfrac{L}{kA}$
2.	Radial flow through a right circular cylinder. 	$R = \dfrac{\log_e \frac{r_o}{r_i}}{2\pi kN}$ (See footnote c).
3.	The buried cylinder. 	$R = \dfrac{\log_e\left(\dfrac{a+\sqrt{a^2+r^2}}{r}\right)}{2\pi kN} = \dfrac{\cosh^{-1}\left(\dfrac{a}{r}\sqrt{1+\dfrac{r^2}{a^2}}\right)}{2\pi kN}$ For $\frac{a}{r} \geqq 3$, satisfactory approximation is: $R \simeq \dfrac{\log_e \frac{2a}{r}}{2\pi kN} = \dfrac{\cosh^{-1}\frac{a}{r}}{2\pi kN}$
4.	Radial flow in a hollow sphere. 	$R = \dfrac{\frac{1}{r_i} - \frac{1}{r_o}}{4\pi k}$
5.	The straight fin or rod heated at one end. 	$R = \dfrac{m}{h_s p \tanh mL}$ (see footnotes d and e). For $ml > 2.3$, $\tanh mL \approx 1$ $m = \sqrt{h_s p/kA}$ A = conduction cross-section area. p = perimeter of cross-section A. h_s = unit conductance to the surroundings from the fin surface. k = thermal conductivity fin material. Δt = wall temperature—ambient temperature.
6.	Finned surface of area HB. 	$R = \dfrac{(s+\delta)}{h_s\left(\dfrac{2}{m}\tanh ml + s\right)HB}$ $m = \sqrt{\dfrac{h_s p}{kA}} = \sqrt{\dfrac{2h_s}{k\delta}}$ Δt defined as in Case 5 above.

[a] The dimensions to be employed in these solutions are: *length of dimension p, L, r* = *feet*; units of k = Btu per (hour) (square foot) (Fahrenheit degree for *one foot thickness*); units of h, Btu per (hour) (square foot) (Fahrenheit degree); units of area, A = square feet.

[b] The thermal conductivity, k, in these solutions should be taken at the average material temperature.

[c] $\log_e x = 2.303 \log_{10} x$.

[d] This expression can *also* be employed *as an approximation for* tapered fins or of annular fins by employing average magnitudes of A and p.

[e] tanh is the hyperbolic tangent.

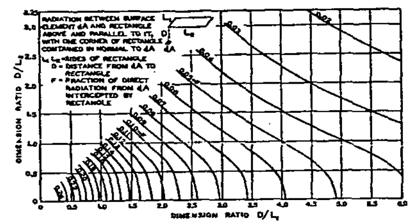


Fig. 4 . . . . Geometrical Factor F for Direct Radiation Between an Element dA and a Parallel Rectangle*

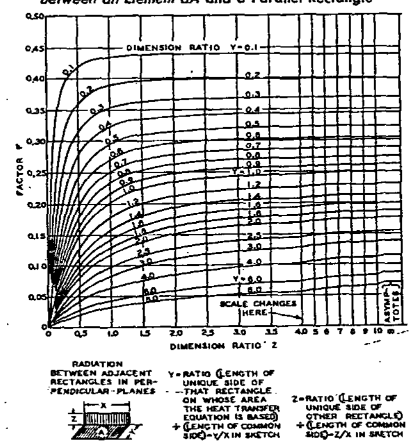


Fig. 5 . . . . Geometrical Factor F for Direct Radiation Between Adjacent Rectangles in Perpendicular Planes*

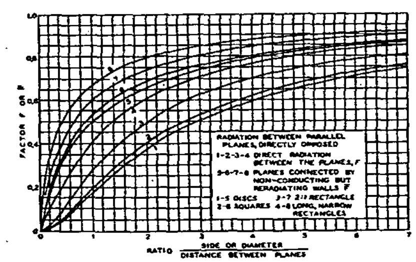


Fig. 6 . . . . Geometrical Factor F for Direct Radiation Between Opposed Parallel Rectangle and Discs of Equal Size*

* H. C. Hottel, Radiant heat transmission, (*Mechanical Engineering*, July 1930, pp. 700 to 702).

convection between the water and the pipe wall. The equation for this case is the following:

$$h_c = 13.9(t_f)^{0.542}\frac{u_m^{0.8}}{D^{0.2}} \quad (17)$$

where

$$u_m = 5 \text{ fps}$$

$$D = \frac{2.067}{12} = 0.1725 \text{ ft}$$

$$t_f = \frac{34 + 36}{2} = 35 \text{ F}$$

$$h_c = \frac{13.9 \times 6.9 \times 3.62}{0.703} = 494 \text{ Btu per (hr) (sq ft) (F deg).}$$

This heat transfer rate is through the inner surface of the pipe and it is, therefore, this area that determines the resistance R_1.

$$A = \pi D = 0.542 \text{ sq ft per unit length of pipe,}$$

and therefore

$$R_1 = \frac{1}{h_c A} = \frac{1}{494 \times 0.542}$$

$$R_1 = 3.73 \times 10^{-3} \text{ (hr) (F deg) per Btu.}$$

Case 11 of Table 2 fits the conditions of the problem if only free convection heating of the pipe is assumed. The equation in this case is as follows:

$$h_c = 0.271\left(\frac{P}{P_o}\right)^{0.5}\left(\frac{\Delta t}{D}\right)^{0.25} \quad (18)$$

where

$$\Delta t = 20 \text{ F}$$
$$D = 0.364 \text{ ft}$$
$$P = P_o = \text{one atmosphere}$$

therefore,

$$h_c = 0.271\left(\frac{20}{0.364}\right)^{0.25}$$

$$h_c = 0.737 \text{ Btu per (hr) (sq ft) (F deg)}$$

Using the surface area of the insulation, the value of the resistance per unit length is determined.

$$A = \pi\left(\frac{4.375}{12}\right) \times 1 = 1.14 \text{ sq ft}$$

$$R_e = \frac{1}{h_c A} = \frac{1}{0.737 \times 1.14}$$

$$R_e = 1.19 \text{ (hr) (F deg) per Btu.}$$

This result may not be deemed conservative inasmuch as the expression is for still air. If, however, the air is not still, but flows at approximately 5 mph or 7 fps, the heat transfer equation for forced convection would apply. This equation is Case 5 of Table 2.

$$h_{c(average)} = 0.211(T_f)^{0.43}\frac{(u_a\rho)^{0.8}}{D^{0.4}} \quad (19)$$