The humidity ratio W is the mol ratio n_w/n_a times the ratio of molecular weights, namely, $18.016/28.966 = 0.6220$; hence (13) can be written

$$W = 0.6220 \frac{p_w}{p - p_w} \tag{14}$$

Now, even if it is assumed that both the dry air and the water vapor behave like perfect gases, it does not follow that at saturation the "partial" pressure of the water vapor can be put equal to the saturation pressure of pure water at the temperature of the mixture because; (1) the coexisting liquid (or solid) phase is not pure water but contains a small amount of dissolved air, and (2) the coexisting liquid (or solid) phase has to support the observed pressure p and not just the saturation pressure p_s of pure water. These effects are calculable but are in general smaller than the effects of intermolecular forces which have already been ignored. Besides, the only legitimate reason for retaining Dalton's Rule is to gain simplicity; hence these effects should be disregarded also, and the humidity ratio at saturation estimated as follows,

$$W_s = 0.6220 \frac{p_s}{p - p_s} \tag{15}$$

In this chapter the ratio W/W_s has been called degree of saturation and denoted by the greek letter μ. The ratio p_w/p_s has long been called *relative humidity* and will be denoted by the Greek letter φ. Equations 14 and 15 can be combined to give

$$\mu = \varphi \frac{1 - p_s/p}{1 - \varphi p_s/p} \tag{16}$$

which can be inverted to give

$$\varphi = \frac{\mu}{1 - (1 - \mu)p_s/p} \tag{17}$$

Example 16. Find the relative humidity of moist air at 180 F, 20 per cent saturation.

Solution. Inserting numerical data from Table 1 into (17), the answer is

$$\varphi = \frac{0.20}{1 - 0.80 \times 15.294/29.921} = 0.3384$$

Example 17. Find the degree of saturation of moist air at 70 F, 50 per cent relative humidity.

Solution. Inserting numerical data from Table 1 into Equation 16, the answer is

$$\mu = 0.50 \frac{1 - 0.73915/29.921}{1 - 0.50 \times 0.73915/29.921} = 0.4937$$

The foregoing examples show that there is a substantial difference between degree of saturation and relative humidity, particularly at higher temperatures. Of course, they both have the value zero for dry air and the value unity for saturated moist air regardless of the temperature.

A Dalton Rule expression for the volume of moist air per pound of dry air obtainable directly from Equation 12 is

$$v = \left(\frac{R_a T}{p}\right) + \mu \left(\frac{W_s R_w T}{p}\right) \tag{18}$$

where

R_a = gas constant for dry air = 1545.31 ÷ 28.966 = 53.349 (ft/F).

R_w = gas constant for water vapor = 1545.31 ÷ 18.016 = 85.774 (ft/F).

This expression is of the form of Equation 4.

According to Dalton's Rule the enthalpy of moist air is the sum of separate contributions from the dry air and the water vapor; thus,

$$h = h_a + \mu(W_s h_w) \tag{19}$$

where, to be consistent, the specific enthalpies h_a and h_w should be allowed to vary with temperature only, not with pressure or composition. This expression is of the form of Equation 5.

Within the accuracy of Dalton's Rule the following empirical equations give suitable values of h_a and h_w:

$$h_a = 0.240t$$
$$h_w = 0.444t + 1061 \tag{20}$$

Equation 7 defining thermodynamic wet-bulb temperature may be written in the form,

$$h - h' + (W_s^* - W)h_w^* = h_s^* - h'$$

If the quantity h' that has been subtracted from both sides is understood to be the enthalpy at the thermodynamic wet-bulb temperature t^* but at the humidity ratio W, then within the accuracy of Dalton's Rule

$$h - h' = (0.240 + 0.444W)(t - t^*)$$
$$h_s^* - h' = (1061 + 0.444t^*)(W_s^* - W)$$
$$h_w^* = t^* - 32$$

With these approximations Equation 7 becomes

$$W_s^* - W = \frac{0.240 + 0.444W}{1093 - 0.556t^*}(t - t^*) \tag{21}$$

Carrier [6] has modified Equation 21 by introducing further approximations as follows,

$$W_s^* = 0.6220p_s^*/(p - p_s^*)$$
$$W = 0.6220p_w/(p - p_s^*)$$
$$0.444\,W = 0$$

the first of which is part of Dalton's Rule. The result is

$$p_w = p_s^* - \frac{p - p_s^*}{2830 - 1.44t^*}(t - t^*) \tag{22}$$

except that the numerical values of the constants in the denominator of the rightmost term are somewhat different than Carrier's.

Equation 22 permits direct calculation of the partial pressure p_w from observed values of pressure p, temperature t, and wet-bulb temperature t^*, assuming that information is available regarding the saturation pressure p_s. The ratio p_w/p_s is the so-called relative humidity.

Example 18. Find the relative humidity of moist air at 90 F dry-bulb, and 63 F (thermodynamic wet-bulb).

Solution. At 63 F the value of the saturation pressure is 0.58002 in. Hg. Therefore, at atmospheric pressure (29.921 in. Hg),

$$p_w = 0.58002 - 29.341 \times 27/2739 = 0.2908 \text{ in. Hg}$$