It is convenient in the analysis of compressible flow to introduce the velocity of propagation of pressure impulses or, more familiarly, the sonic velocity, a. For perfect gases this is given by the equation:

$$a^2 = kgp/\rho = kgRT \tag{22}$$

Accordingly, Equation 21 may be written

$$\frac{1}{2}(V_2^2 - V_1^2) + \frac{a_1^2}{k-1}\left[\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} - 1\right] = 0 \tag{23}$$

or, by rearrangement,

$$\frac{p_2}{p_1} = \left[1 - \frac{k-1}{a_1}\left(\frac{V_2^2 - V_1^2}{2}\right)\right]^{\frac{k}{k-1}} \tag{24}$$

which permits the calculation of the ratio of pressures at entrance and exit of the steady flow device—pipe, orifice, or nozzle. From Equations 19 and 22 it follows that

$$\frac{a_2^2}{a_1^2} = \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} \tag{25}$$

so that

$$\frac{k-1}{2}(V_2^2 - V_1^2) + a_2^2 - a_1^2 = 0 \tag{26}$$

and

$$\frac{a_2^2}{a_1^2} = \frac{1 + \frac{k-1}{2}\frac{V_1^2}{a_1^2}}{1 + \frac{k-1}{2}\frac{V_2^2}{a_2^2}} \tag{27}$$

The ratio of flow velocity to sonic velocity is known as the Mach number,

$$M = V/a$$

This parameter is particularly useful in compressible flow analysis. In general, if $M \leqq 0.1$ the flow may be considered to be incompressible. This is generally true in heating and ventilating air ducts.

In terms of the Mach number.

$$\frac{a_2^2}{a_1^2} = \frac{T_2}{T_1} = \frac{1 + \frac{k-1}{2}M_1^2}{1 + \frac{k-1}{2}M_2^2} \tag{28}$$

and

$$\frac{p_2}{p_1} = \left(\frac{1 + \frac{k-1}{2}M_1^2}{1 + \frac{k-1}{2}M_2^2}\right)^{\frac{k}{k-1}} \tag{29}$$

The quantity,

$$p^0 = p\left(1 + \frac{k-1}{2}M^2\right)^{\frac{k}{k-1}} \tag{30}$$

is called the stagnation pressure, and gives a measure of pressure energy. For incompressible flow

$$p^0 = p + \frac{1}{2g}\rho V^2 = p + q \tag{31}$$

where

$$q = \frac{1}{2g}\rho V^2 \tag{32}$$

and is the dynamic pressure. A total head tube measures stagnation pressure directly.

From Equation 29 it follows that for frictionless, adiabatic flow

$$p_2^0 = p_1^0 \tag{33}$$

This represents another extension of the Bernoulli equation to compressible flow. Friction will cause a loss in pressure energy.

Ideal Flow through Nozzle or Orifice

The majority of low measuring systems depend upon a correlation between pressure drop, area, and quantity of flow. The basic formulas may be stated on the assumption that the flow is frictionless and adiabatic. Designating the main stream by station 1, and flow at some measuring restriction by station 2, the flow in pounds per second is

$$w = \rho_1 A_1 V_1 = \rho_2 A_2 V_2 \tag{34}$$

or, in terms of Mach number,

$$w = A_2 M_2 \sqrt{kgp_2\rho_2} \tag{35}$$

According to Equation 29

$$\left(\frac{p_1}{p_2}\right)^{\frac{k-1}{k}} = \frac{1 + \frac{k-1}{2}M_2^2}{1 + \frac{k-1}{2}M_1^2} \tag{36}$$

from which

$$M_2^2 = \frac{2}{k-1}\left(\frac{1 + \frac{k-1}{2}M_1^2}{1 - M_1^2/M_2^2}\right)\left[\left(\frac{p_1}{p_2}\right)^{\frac{k-1}{k}} - 1\right] \tag{37}$$

so that

$$w = A_2\sqrt{\frac{1 + \frac{k-1}{2}M_1^2}{1 - M_1^2/M_2^2}}\sqrt{\frac{2kgp_2\rho_2}{k-1}\left[\left(\frac{p_1}{p_2}\right)^{\frac{k-1}{k}} - 1\right]} \tag{38}$$

If the initial velocity is sufficiently small, M_1^2 will be negligible so that

$$w = A_2\sqrt{\frac{2kgp_2\rho_2}{k-1}\left[\left(\frac{p_1}{p_2}\right)^{\frac{k-1}{k}} - 1\right]} = A_2\sqrt{\frac{2kgp_1\rho_1}{k-1}\left(\frac{p_2}{p_1}\right)^{\frac{k+1}{k}}\left[\left(\frac{p_1}{p_2}\right)^{\frac{k-1}{k}} - 1\right]} \tag{39}$$

If this is computed and the figures are plotted, the curved line (partly