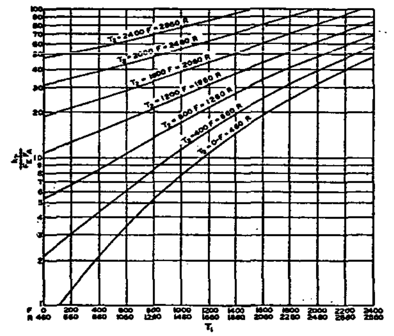

Fig. 7 .... Equivalent Conductance for Radiation Between Two Black Bodies Exchanging Energy Only with One Another

$$T_f = \frac{100 + 120}{2} + 460 = 570 \text{ Rankine (F absolute)}$$

$$u_m = 7 \text{ fps}$$

$$\rho = 0.076 \left(\frac{520}{570}\right) = 0.0694 \text{ lb per cu ft}$$

$$D = 0.364 \text{ ft}$$

$$h_{c(average)} = \frac{0.211(570)^{0.42}(7 \times 0.0694)^{0.8}}{(0.364)^{0.4}}$$

$$h_{c(average)} = 2.73 \text{ Btu per (hr) (sq ft) (F deg)}$$

and

$$R_c \text{ (Forced Convection)} = \frac{1}{h_c A} = \frac{1}{2.73 \times 1.14}$$

$$R_c = 0.321 \text{ (hr) (F deg) per Btu.}$$

The radiation resistance, R_r, which acts in parallel with the resistance just calculated, can be computed with the aid of Fig. 7. The pipe wall, assumed at 100 F sees the surroundings at 120 F. If these two temperatures are used with Fig. 7, a value for $\frac{h_r}{F_A F_E}$ is determined directly.

$$\therefore \frac{h_r}{F_A F_E} = 1.4 \text{ Btu per (hr) (F deg) (sq ft).}$$

The angle factor, F_A, is unity, and for an estimated surface emissivity of 0.95 (see Table 3), $F_E = 0.95$. Therefore,

$$h_r = 1.4\, F_A F_E = 1.4 \times 1 \times 0.95$$
$$h_r = 1.33 \text{ Btu per (hr) (F deg) (sq ft)}$$

and the radiation resistance, R_r, is then the following:

$$R_r = \frac{1}{h_r A} = \frac{1}{1.33 \times 1.14}$$

$$R_r = 0.659 \text{ (hr) (F deg) per Btu.}$$

The resultant resistance of R_c and R_r acting in parallel (see Fig. 8) can now be evaluated as:

$$\frac{1}{R_4} = \frac{1}{R_c} + \frac{1}{R_r} = \frac{1}{0.321} + \frac{1}{0.659} = 4.54 \text{ Btu per (hr) (F deg)}$$

$$R_4 = 0.216 \text{ (hr) (F deg) per Btu.}$$

The overall resistance, R_T, surroundings to cold water, is the sum of $R_1 + R_2 + R_3 + R_4 = 4.12$ (hr) (F deg) per Btu for 1-ft length of pipe. Note that the controlling resistances are R_3 and R_4, and that neglect of both R_1 and R_2 would not significantly influence the total resistance, R_T.

On the basis of this resistance calculation, the heat transfer from the surroundings to the cold water may be evaluated as:

$$\frac{q_{rc}}{N} = \frac{t_e - t_f}{R_T} = \frac{120 - 34}{4.12} = 20.8 \text{ Btu per (hr) (ft)}$$

or about 0.175 tons of refrigeration per 100 ft of pipe.

Since the calculation is based on a 1-ft pipe length,

$$q_{rc} = 20.8 \text{ Btu per hr.}$$

The temperature drops through the various resistances are now readily evaluated by Equation 16 as:

$$\Delta t = qR$$

$t_e - t_{s3}$ (air to insulation surface)
$$= qR_4 = 20.8 \times 0.216 = 4.49 \text{ F deg}$$

$t_{s3} - t_{s2}$ (through the insulation)
$$= qR_3 = 20.8 \times 3.9 = 81.2 \text{ F deg}$$

$t_{s2} - t_{s1}$ (through the pipe wall)
$$= qR_2 = 20.8 \times 8.5 \times 10^{-4} = 0.018 \text{ F deg}$$

$t_{s1} - t_f$ (pipe wall to cold water)
$$= qR_1 = 20.8 \times 3.73 \times 10^{-3} = 0.078 \text{ F deg}$$

This solution was obtained on the temperature distribution assumptions initially made. It is apparent that a better solution could be obtained if the whole problem were reiterated using the temperature distribution just calculated.

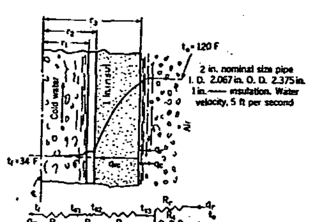

Fig. 8 .... Heat Transfer Conditions in an Insulated Cold Water Line

PERIODIC AND TRANSIENT HEAT FLOW

The foregoing data and examples dealt with steady-state heat transfer (not varying with time). In most practical heat transfer problems the heat flow depends upon time. Such cases can usually be divided into two classes: periodic and transient. Periodic heat transfer repeats periodically in time. Transient heat transfer exhibits no periodicity. Graphical, analytical, and numerical methods are available for solving transient or periodic heat flow problems.[4,9,10,11,12] Graphical and numerical methods are the most versatile, and can be applied with minimum mathematical training.

A large number of analytical solutions for the case of heat conduction in variously shaped solids are available in the literature. Table 7 gives a summary of the cases reported and tabulated. Many more analytical solutions are available in the form of infinite series,[11,12,13,19] but are not tabulated. Certain complex cases may be treated by combining the simple analytical solutions as discussed in Reference 17. (See also Reference 20.)

Frequently, transient heat flow problems in one dimension have boundary conditions which make the problem difficult to treat analytically. In such cases, recourse may be made to a graphical method of solution sometimes called the Schmidt method.[4,10,19,21,22] This method will be briefly outlined for the case of transient heat flow in a slab insulated on one face, and suddenly exposed on the other face through a fixed thermal resistance to a higher temperature. The technique is general, however, and methods may be devised for any boundary conditions,[9,22] and also, for one dimensional (radial) heat flow in spheres and cylinders.[22,23]

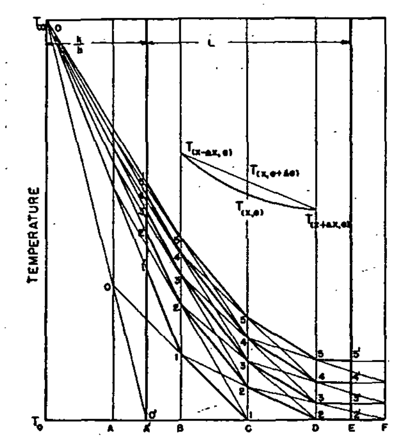

Fig. 9 .... Example of a Graphical Solution to a Problem in Transient Heat Conduction

Consider the slab to be divided, as shown in Fig. 9, by n equidistant planes parallel to the slab surface and a distance Δx apart. Let the temperature of the slab at any plane and any time (θ) be denoted by $T_{x,\theta}$. Then the temperature of the slab at the two adjacent planes at the same time will be denoted as $T_{(x+\Delta x,\theta)}$ and $T_{(x-\Delta x,\theta)}$. In a similar manner the temperature of the x plane at a time $\Delta\theta$ later will be $T_{(x,\theta+\Delta\theta)}$.

In accordance with this nomenclature, the temperature at any plane x and time $\theta + \Delta\theta$ is given as

$$T_{(x,\theta+\Delta\theta)} = \frac{T_{(x+\Delta x,\theta)} + T_{(x-\Delta x,\theta)}}{2} \qquad (20)$$

which may be interpreted as follows. The temperature of the slab at any plane, x, and any time, θ, is equal to the average temperature of the two adjacent planes obtained at the time ($\theta - \Delta\theta$).

The time interval $\Delta\theta$ is determined by the equation

$$\Delta\theta = \frac{\Delta x^2}{2\alpha} \qquad (21)$$

Omitting the graphical construction at the slab boundaries, reference to Fig. 9 demonstrates the graphical method by means of which the temperature at each plane is determined at successive intervals of time in accordance with Equation 20.

For the problem stated, the boundary condition at the insulated surface is specified by the equation

$$\frac{\partial T}{\partial x} = 0$$

and at the uninsulated face by the equation

$$h(T_\infty - T) = -k\frac{\partial T}{\partial x}$$

In terms of finite differences these two equations (employing nomenclature established by Fig. 9) become

$$\frac{T_F - T_E}{\Delta x} = 0 \quad \text{or} \quad T_F = T_E \quad \text{at} \quad x = L$$

and

$$h(T_\infty - T_{A'}) = -k\frac{(T_B - T_A)}{\Delta x} \quad \text{at} \quad x = 0$$

or

$$\frac{T_\infty - T_{A'}}{k/h} = -\frac{T_A - T_B}{\Delta x}$$

The details of the graphical construction are best obtained by inspection of Fig. 9. Note that the line (0,0,0′) used to initiate the graphical construction, is the only one drawn to the slab boundary A'. The numbered points indicate temperatures at the sub-slab boundaries at 1,2,3, etc., time intervals ($\Delta\theta$) after the slab is exposed to the high temperature.

For transient heat flow in two dimensions, and also, for steady-state conduction, numerical methods of solution are available in the literature.[9,10,12,19] These numerical methods are applicable to three-dimensional problems, although the calculations involved normally become too tedious for most applications of the method. An additional technique of solution for one- and two-dimensional problems in transient conduction results from the analogy of electrical resistance-capacitance networks to thermal systems.[24]

For two-dimensional problems in steady state conduction, additional techniques of solution are found in flux plotting,[9,10] and in the use of a potential tank.[9,19] These methods are of