

- R = gas constant for air, = 53.3 ft.-lb. per (lb) (Rankine degree).
- r_c = compressor (or fan) pressure ratio (greater than 1.0), dimensionless.
- r_t = turbine pressure ratio (greater than 1.0), dimensionless.
- T = total temperature of air, Rankine.
- T_{ave} = average total temperature of air between two points in the system, Rankine.
- W_b = flow rate of bleed air, pounds per minute.
- W_c = flow rate of cooling air (basic system), pounds per minute.
- W_{c1} = flow rate of cooling air in primary heat exchanger (bootstrap system), pounds per minute.
- W_{c2} = flow rate of cooling air in secondary heat exchanger (bootstrap system), pounds per minute.
- Y_c = compression factor = $r_c^{(\gamma-1)/\gamma}$, dimensionless.
- Y_e = expansion factor = $1/r_t^{(\gamma-1)/\gamma}$, dimensionless.
- ΔP = pressure difference, inches Hg.
- ΔT = temperature difference, Fahrenheit degrees.
- ΔT_i = isentropic temperature difference of air across turbine, Fahrenheit degrees.
- γ = ratio of specific heat at constant pressure to specific heat at constant volume, dimensionless. For air, $\gamma = 1.395$.
- η_c = isentropic efficiency of compressor, dimensionless.
- η_m = mechanical efficiency of cooling turbine
- fan or compressor power
turbine power, dimensionless.
- η_t = isentropic efficiency of turbine, dimensionless.
- ρ = ratio of any air density to standard air density, dimensionless.

Subscripts 1, 2, 3, ... denote locations in the refrigeration systems shown in Figs. 15 and 23.

Example 1: For the basic system and conditions shown in Fig. 15, determine: (a) the flow rate of bleed air W_b , (b) the flow rate of cooling air W_c , and (c) turbine discharge air temperature T_3 . The effective area of the turbine nozzle is 0.328 sq in. Note that all temperatures and pressures are total values unless otherwise stated. Dry air is assumed throughout the analysis.

Solution: A trial-and-error solution is necessary. Values are assumed for the bleed and cooling air flow rates W_b and W_c , and the turbine speed N .

Assume $W_b = 20$ lb per min., $W_c = 56$ lb per min., and $N = 67,000$ rpm.

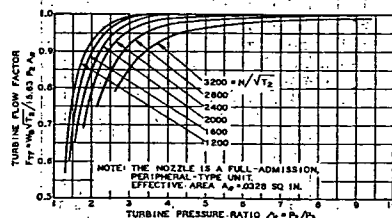


Fig. 12.... Test Performance Curve—Flow Factor vs Pressure Ratio*

Bleed Air Circuit

From Fig. 10, using the assumed values for W_b and W_c , read $E = 0.932$.

$$E = \frac{T_1 - T_2}{T_1 - T_3} = \frac{\Delta T_{1-2}}{T_1 - T_3}$$

$$\Delta T_{1-2} = E(T_1 - T_3) = 0.932(877 - 596) = 262 \text{ F deg}$$

$$T_2 = T_1 - \Delta T_{1-2} = 877 - 262 = 615 \text{ R}$$

The average temperature between points 1 and 2 is

$$(T_{ave})_{1-2} = \frac{T_1 + T_2}{2} = \frac{877 + 615}{2} = 746 \text{ R}$$

From Fig. 11, for $W_b = 20$ lb per min, read $\sigma \Delta P = 12.3$ in. Hg. Since

$$\sigma = \frac{(P_{ave})_{1-2}}{(T_{ave})_{1-2}}$$

where 17.35 is the ratio of standard temperature to standard pressure (519/29.92).

$$\Delta P_{1-2} = \frac{(\sigma \Delta P_{1-2})(T_{ave})_{1-2}}{17.35} = \frac{(12.3)(746)}{17.35} = 5.1 \text{ in. Hg.}$$

(Note that this equation is arranged in a form convenient for solution on a slide rule, but requires a trial-and-error process. It can be rearranged in quadratic form and solved directly for ΔP_{1-2} .)

$$P_2 = P_1 - \Delta P_{1-2} = 108 - 5 = 103 \text{ in. Hg abs.}$$

The turbine pressure ratio $r_t = P_1/P_2 = 103/33 = 3.12$. The expansion factor is

$$Y_e = 1 - \frac{1}{r_t^{(\gamma-1)/\gamma}} = 1 - \frac{1}{(3.12)^{(1.395-1)/1.395}} = 0.275$$

Isentropic temperature drop

$$(\Delta T_i)_{1-2} = T_1 Y_e = 815(0.275) = 169 \text{ F deg.}$$

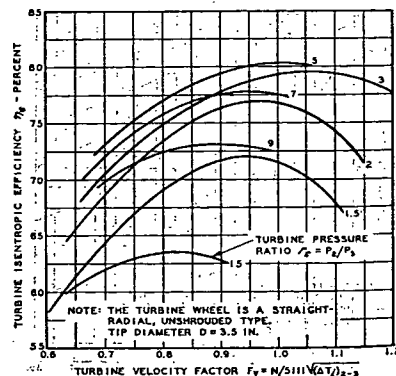
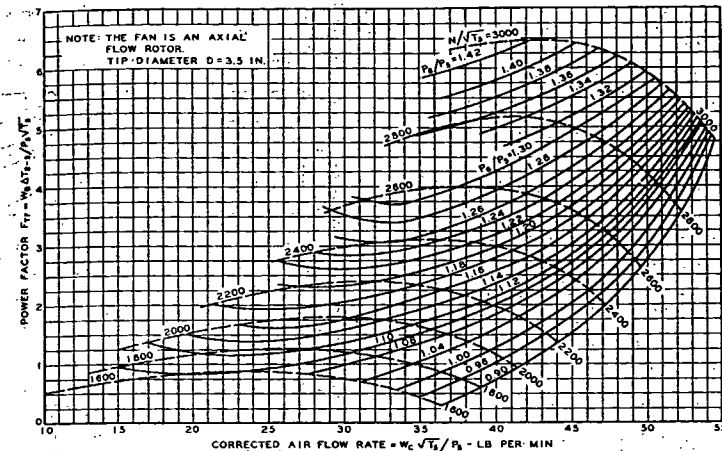


Fig. 13.... Test Performance Curve—Efficiency vs Velocity Factor*



* For Fan of Cooling Turbine in a Basic Air-Cycle Refrigeration Unit

Fig. 14.... Test Performance Curve—Power Factor vs Corrected Air Flow Rate*

For the assumed turbine speed N of 67,000 rpm,

$$N/\sqrt{T_1} = 67,000/\sqrt{615} = 2700.$$

From Fig. 12, read the turbine flow factor $F_{TF} = 0.94$.

$$F_{TF} = \frac{W_b \sqrt{T_1}}{15.63 P_1 A_2}$$

where

$$15.63 = \left[\left(\frac{r_t}{\gamma-1} \right) \left(\frac{2}{\gamma+1} \right)^{(\gamma+1)/(\gamma-1)} \right]^{1/2} \sqrt{\frac{g}{J_c}} \left(\frac{60}{2.036} \right)$$

$$W_b = \frac{15.63 P_1 A_2}{\sqrt{T_1}} (F_{TF}) = \frac{(15.63)(103)(0.328)(0.94)}{\sqrt{615}} = 20.01 \text{ lb per min.}$$

The calculated value of W_b checks with the original assumed value. If the calculated value had not checked, a second trial solution would have been necessary.

The turbine velocity factor

$$F_v = \frac{N}{K \sqrt{(\Delta T_i)_{1-2}}}$$

where

$$K = \sqrt{J_c \rho_1 \left(\frac{12 \times 60}{\pi D} \right)^2}$$

$$= \sqrt{(778)(32.2)(0.24) \left(\frac{720}{3.14 \times 3.5} \right)^2}$$

$$= 5111 \text{ rpm per } \sqrt{\text{R deg.}}$$

Therefore,

$$F_v = \frac{67000}{5111 \sqrt{169}} = 1.007$$

From Fig. 13, read the turbine efficiency $\eta_t = 79.6$ percent.

$$\Delta T_{1-2} = \frac{\eta_t}{100} (\Delta T_i)_{1-2} = \frac{79.6}{100} (169) = 134.5 \text{ R.}$$

The turbine discharge air temperature is

$$T_2 = T_1 - \Delta T_{1-2} = 815 - 134.5 = 680.5 \text{ R (20.5 F).}$$

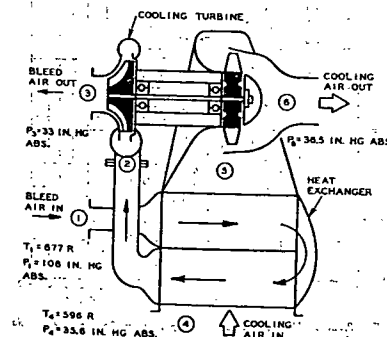


Fig. 15.... Basic Air-Cycle Refrigeration System for Example 1*