

The same resistance analysis may be applied to complicated steady-state conduction problems. Table 8 indicates the solutions in six common cases of steady-state conduction.

A complete analysis by the resistance method is well illustrated by considering the heat transfer from the air outside to the cold water inside of an insulated pipe. The temperature gradients and the nature of the resistance analysis are indicated by the two sketches of Fig. 4.

Since air is sensibly transparent to radiation, there will be some heat transfer by both radiation and convection to the outer insulation surface. The mechanisms act in parallel on the air side. The total current by radiation and convection then passes through the insulating layer and the pipe wall by thermal conduction, and thence by convection into main cold water streams. Radiation is not significant on the water side as liquids are sensibly opaque to radiation, although water transmits energy in the visible region. The contact resistance between the insulation and the pipe wall is presumed to be equal to zero.

Referring to Fig. 4, the thermal current for a given length N of pipe, q_{rc} , Btu per hour, may be thought of as flowing through the parallel resistances R_i and R_c , associated with the insulation surface radiation and convection transfer. Then the flow is through the resistance offered to thermal conduction by the insulation, R_s , through the pipe wall resistance, R_p , and into the water stream through the convection resistance, R_w . Note the analogy to the direct current electrical circuit problem. A temperature (potential) drop is required to overcome these resistances to the flow of thermal current. The total resistance to heat transfer, R_T , hour degrees Fahrenheit per Btu, is the summation of the individual resistances:

$$R_T = R_i + R_s + R_p + R_w \quad (9)$$

where the resultant parallel resistance R_i is obtained from:

$$\frac{1}{R_i} = \frac{1}{R_r} + \frac{1}{R_c}$$

Provided the individual resistances may be evaluated, the total resistance can be obtained from this relation. Then the heat transfer current for the length of pipe (N , ft) can be established by the relation:

$$q_{rc} \text{ (Btu per hour)} = \frac{t_o - t_i}{R_T} \quad (10)$$

For a unit length of the pipe the heat transfer rate is:

$$\frac{q_{rc}}{N} \text{ (Btu per hour foot)} = \frac{(t_o - t_i)}{R_T N} \quad (11)$$

The temperature drop, Δt , through an individual resistance may then be calculated from the relation:

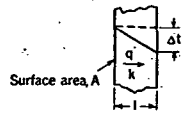
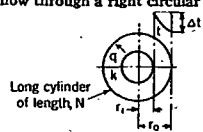
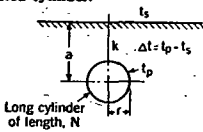
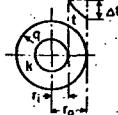
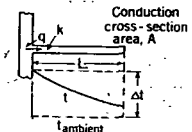
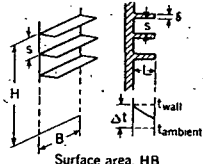
$$\Delta t = R q_{rc}$$

where R is the resistance in question.

The problem is now reduced to one of evaluating the individual resistances of the system. This entails suitable integration of the rate Equations 1, 2 and 3 to produce expressions of the form:

$$q = \frac{\Delta t}{R} \quad (12)$$

TABLE 8. SOLUTIONS FOR SOME STEADY-STATE THERMAL CONDUCTION PROBLEMS^{a,b}

No.	SYSTEM	Expressions for the resistance R entering into the equation: $q = \Delta t/R$ (Btu per hour)
1.	Flat wall or curved wall if curvature is small (wall thickness less than 0.1 of inside diameter). 	$R = \frac{L}{kA}$
2.	Radial flow through a right circular cylinder. 	$R = \frac{\log_e \frac{r_o}{r_i}}{2\pi k N}$ (See footnote c).
3.	The buried cylinder. 	$R = \frac{\log_e \frac{2a}{r}}{2\pi k N}; R = \frac{\cosh^{-1} \frac{a}{r}}{2\pi k N}$ for $\frac{a}{r} \geq 3$ (See footnote c).
4.	Radial flow in a hollow sphere. 	$R = \frac{1}{4\pi k} \left(\frac{1}{r_i} - \frac{1}{r_o} \right)$
5.	The straight fin or rod heated at one end. 	$R = \frac{m}{h_a p \tanh mL}$ (see footnotes d and e). For $mL > 2.3$, $\tanh mL \approx 1$ $m = \sqrt{h_a p / kA}$ A = conduction cross-section area. p = perimeter of cross-section A . h_a = unit conductance to the surroundings from the fin surface. k = thermal conductivity fin material. Δt = wall temperature—ambient temperature
6.	Finned surface of area HB . 	$R = \frac{(s + \delta)}{h_a \left(\frac{2}{m} \tanh mL + s \right) HB}$ $m = \sqrt{\frac{h_a p}{kA}} = \sqrt{\frac{2 h_a}{ks}}$ Δt defined as in Case 5 above.

^aThe dimensions to be employed in these solutions are: length of dimension p, L, r = feet; units of k = Btu per (hour) (square foot) (degree Fahrenheit for one foot thickness); units of h, h_a , Btu per (hour) (square foot) (degree Fahrenheit); units of area, A = square feet.

^bThe thermal conductivity, k , in these solutions should be taken at the average material temperature (see Table 5).

^c $\log_e x = 2.303 \log_{10} x$.

^dThis expression can also be employed as an approximation for tapered fins or of annular fins by employing average magnitudes of A and p .

^eTanh is the hyperbolic tangent.