tube or longitudinal fins inside the tubes. Since the heat transfer coefficient for the refrigerant side is generally smaller than the coefficient for the water side, the addition of fins on the refrigerant side leads to compact designs and to appreciable improvement in performance.

When fins are inside the tubes, they tend to spread the liquid refrigerant over a larger area, thus giving a film of small thickness which results in higher heat transfer coefficients. High fin efficiency is important for boiling, since the coefficient is reduced by lower temperature difference, especially, if this shifts boiling from the nucleate regime to the free convection regime (Fig. 7).

When fins are inside tubes, the refrigerant pressure drop can be high. This will result in reduced available temperature difference and lower heat transfer. It can be avoided by carefully determining the number of parallel refrigerant passes which give optimum loading for best overall heat transfer.

OVERALL HEAT TRANSFER

In most of the steady state heat transfer problems encountered in practice, more than one of the heat transfer modes may be involved simultaneously. Therefore, it is convenient to combine the various heat transfer coefficients into an overall coefficient in order that the total heat transfer may be calculated from the terminal temperatures. The solution to this problem is made much simpler if one employs the concept of the *thermal circuit* and *thermal resistance*.

Local Overall Coefficient of Heat Transfer—Resistance Method

Consider an application where heat is transferred from one fluid to another by a three-step steady-state process: from a warmer fluid to a solid wall, through the solid wall, and thence to a colder fluid. It is customary to employ an *overall coefficient* of heat transfer, U, based on the overall difference between the bulk temperatures of the two fluids, $t_1 - t_2$, defined as follows:

$$q = UA(t_1 - t_2) \tag{22}$$

where A is the surface area. Since Equation 22 is essentially a definition of U, the surface area A upon which U is based is arbitrary, and should always be specified in referring to U.

The temperature drops across each part of the heat flow path are as follows:

$$t_1 - t_{s1} = qR_1$$
$$t_{s1} - t_{s2} = qR_2$$
$$t_{s2} - t_2 = qR_3$$

where t_{s1} and t_{s2} are the surface temperatures of the wall on the warm side and cold side respectively, and R_1, R_2, and R_3 are the thermal resistances. Since the same quantity of heat flows through each thermal resistance, these equations can be combined to give:

$$\frac{t_1 - t_2}{q} = \frac{1}{UA} = R_1 + R_2 + R_3 \tag{23}$$

As demonstrated above, the equations involved are analogous to those for electrical circuits; i.e., when there is a thermal current flowing through several resistances in *series*, the resistances are additive.

$$R_o = R_1 + R_2 + R_3 + \cdots + R_n \tag{24}$$

Similarly, *conductance* is the reciprocal of resistance, and for heat flow through several resistances in *parallel*, the conductances are additive:

$$C = \frac{1}{R_o} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n} \tag{25}$$

For conduction through a solid material, the thermal resistance is a function of the mean length of the heat flow path, the thermal conductivity of the material, and the mean cross-sectional area normal to the flow, Equation 2. Equations for various configurations are given in Table 3.

In the case of convection, the thermal resistance is inversely proportional to the convection coefficient, h_c, and the surface area to which it applies.

$$R_c = \frac{1}{h_c A} \tag{26}$$

The thermal resistance for radiation is written similarly to that for convection.

$$R_r = \frac{1}{h_r A} \tag{27}$$

The term *radiation coefficient* h_r has no physical significance as such, being introduced primarily for convenience in computations. In general, it will be a very complex function of the temperatures, radiation properties, and geometrical arrangement of the enclosure and the body in question. For the simple case of a small body in a large enclosure, the radiation coefficient can be determined from Equation 12 as follows:

$$h_r = \frac{q_1/A_1}{T_1 - T_2} = \epsilon_1\sigma(T_1^2 + T_2^2)(T_1 + T_2) \tag{28}$$

where

ϵ_1 = *emissivity of the enclosed body.*
T_1 = absolute temperature of enclosed body.
T_2 = absolute temperature of enclosure.

Equation 28, with $\epsilon_1 = 1$, is plotted in Fig. 20.

The use of these relations for resistance and conductance makes possible the solution of many practical heat transfer problems. As discussed in Chapter 24, the practical analyses of heat transfer in building walls and pipe coverings are usually computed by this method.

A complete analysis by the resistance method is well illustrated by considering the heat transfer from the air outside to the cold water inside an insulated pipe. The temperature gradients and the nature of the resistance analysis are indicated by the two sketches of Fig. 21.

Since air is sensibly transparent to radiation, there will be some heat transfer by both radiation and convection to the outer insulation surface. The mechanisms act in parallel on the air side. The total transfer by radiation and convection then passes through the insulating layer and the pipe wall by thermal conduction, and thence by convection and radiation into the cold water stream. (Radiation is not significant on the water side as liquids are sensibly opaque to radiation, *although water transmits energy in the visible region.*) The contact resistance between the insulation and the pipe wall is assumed negligible.

Referring to Fig. 21, the heat transferred for a given length L of pipe, q_{rc}(Btu/hr) may be thought of as flowing through the parallel resistances R_r and R_c, associated with the insulation surface radiation and convection coefficients. The flow then proceeds through the resistance offered to thermal conduction by the insulation R_3, through the pipe wall resistance R_2, and into the water stream through the convection resistance R_1. Note the analogy to the direct current electrical circuit problem. A temperature (potential) drop is required to overcome these resistances to the flow of thermal current. The total or overall resistance to heat transfer R_o is the summation of the individual resistances:

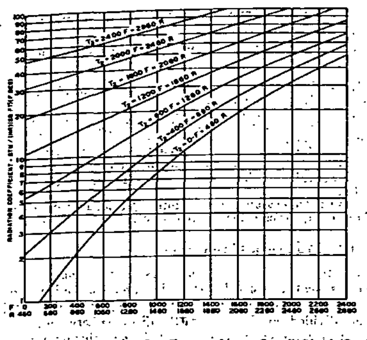

Fig. 20 .... Radiation Coefficient for a Small Black Body in a Large Enclosure

$$R_o = R_1 + R_2 + R_3 + R_4 \tag{29}$$

where the resultant parallel resistance R_4 is obtained from

$$\frac{1}{R_4} = \frac{1}{R_r} + \frac{1}{R_c} \tag{30}$$

Provided the individual resistances may be evaluated, the total resistance can be obtained from this relation. The heat transfer for the length of pipe, L, (ft) can be established by the relation:

$$q_{rc}(\text{Btu/hr}) = (t_a - t)/R_o \tag{31}$$

For a unit length of the pipe the heat transfer rate is

$$q_{rc}/L(\text{Btu/hr ft}) = (t_a - t)/R_oL \tag{32}$$

The temperature drop, Δt, through an individual resistance may then be calculated from the relation:

$$\Delta t = Rq_{rc} \tag{33}$$

where R is the resistance in question.

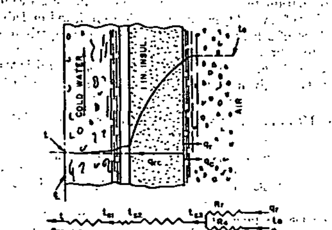

Fig. 21 .... Thermal Circuit Diagram for an Insulated Cold Water Line

Mean Temperature Difference

When heat is exchanged between two fluids flowing continuously through a heat exchanger the local temperature difference, Δt, varies along the flow path. Two methods may be used to account for this variation. In the first, the heat transfer is calculated using the familiar rate equation:

$$q = UA\Delta t_m \tag{34}$$

where U is the overall coefficient of heat transfer from fluid to fluid, A is an area associated with the coefficient U, and Δt_m, is the necessary mean temperature difference.

For parallel flow or counter-flow heat exchangers and for any exchanger in which one of the fluid's temperature is substantially constant, the mean temperature difference is:

$$\Delta t_m = \frac{\Delta t_1 - \Delta t_2}{\log_e \dfrac{\Delta t_1}{\Delta t_2}} = \frac{\Delta t_1 - \Delta t_2}{2.3 \log_{10} \dfrac{\Delta t_1}{\Delta t_2}} \tag{35}$$

where Δt_1 and Δt_2 are the temperature differences between the fluids at inlet and exit of the heat exchanger. Δt_m is often called the *logarithmic mean temperature difference.*

Equation 35 for Δt_m is strictly true only if the overall coefficient and the specific heats of the fluids are constant through the heat exchanger, and there are no heat losses. In most practical cases, these assumptions are reasonably well approximated. A procedure for dealing with cases with a variable overall coefficient U is outlined by McAdams.[9]

Calculations using Equation 34 and Δt_m are found convenient when the terminal temperatures are known. In many cases, however, the temperatures of the fluids leaving the exchanger are not known. To avoid trial-and-error calculations, an alternate method, originally proposed by Nusselt, has been recommended by London and Kays.[44] It involves the use of 3 non-dimensional parameters, defined as follows:

1. *Exchanger Heat Transfer Effectiveness* (η).

$$\eta = \frac{C_h(t_{hi} - t_{ho})}{C_{min}(t_{hi} - t_{ci})} = \frac{C_c(t_{co} - t_{ci})}{C_{min}(t_{hi} - t_{ci})} \tag{36}$$

where

$C_h = (wc_p)_h$ = the hot fluid capacity rate, Btu per (hour) (Fahrenheit degree).
$C_c = (wc_p)_c$ = the cold fluid capacity rate, Btu per (hour) (Fahrenheit degree).
C_{min} = the smaller of the two capacity rates.
t_h = terminal temperature of hot fluid, Fahrenheit degrees. Subscript i indicates entering condition, and subscript o indicates leaving condition.
t_c = terminal temperature of cold fluid, Fahrenheit degrees.

2. *Number of Exchanger Heat Transfer Units* (NTU).

$$(\text{NTU}) = \frac{AU_{avg}}{C_{min}} = \frac{A}{C_{min}}\int_A U(dA) \tag{37}$$

where A is the same heat transfer area used in defining the overall coefficient U.

3. *Capacity Rate Ratio* (Z).

$$Z = \frac{C_{min}}{C_{max}} \tag{38}$$

In general it is possible to express the heat transfer effectiveness for a given exchanger as a function of the number of transfer units and the capacity rate ratio:

$$\eta = f(\text{NTU}, Z, \text{flow arrangement}) \tag{39}$$