where q is the heat transfer rate, and Δt is the potential drop or temperature difference through the resistance R. Table 8 lists such solutions for six different conduction systems. Table 2 in Chapter 6 and Table 1 of this chapter indicate the magnitudes of the thermal conductivities, k, to be employed in the expressions of Table 8, after dividing k by 12.

The solution applicable to the problem depicted in Fig. 4, for the calculation of R_2 and R_3, is case 2 in Table 8. Thus for a *1 ft length* of 2 in. nominal size pipe (I. D. = 2.067 in., O. D. = 2.375 in.) insulated with 1 in. of material having a conductivity of 0.025:

$$R_2 = \frac{\log_e \frac{1.188}{1.033}}{2\pi \times 26 \times 1} = 8.5 \times 10^{-4} \text{ hr Fahrenheit degree per Btu.}$$

$$R_3 = \frac{\log_e \frac{2.188}{1.188}}{2\pi \times 0.025 \times 1} = 3.9 \text{ hr Fahrenheit degree per Btu.}$$

The convection resistances to heat transfer from the pipe wall to the cold water, R_1, and from the air to the surface of the insulating material, R_c, are dependent on the flow conditions prevailing at these surfaces, and on the thermal properties of the fluids. The unit conductances for thermal convection, h_c, Btu per (hour) (square foot) (Fahrenheit degree), have been determined by test for many flow systems. These data may be employed to predict the conductances for *similar flow systems*. Table 5 summarizes some empirical equations expressing such test results.

For the problem under consideration (Fig. 4), case 3 of Table 5 is applicable for the calculation of the cold water side convection resistance R_1. Corresponding to the water velocity of 5 fps, the mass velocity is:

$$G = 5 \text{ (ft per sec)} \times 62.4 \text{ (lb per cu ft)} \times 3600 \text{ (sec per hr)} = 11.2 \times 10^5 \text{ lb per (hour) (square foot).}$$

The inside diameter of the pipe D is $2.067/12 = 0.1725$ ft.

The average water film temperature will be estimated as 36 F (mixed mean fluid temperature of 34 F). Then case 3, Table 5 yields:

$$h_c = 0.00486\,(1 + 0.36)\,\frac{(11.2 \times 10^5)^{0.8}}{(0.1725)^{0.2}} = 650 \text{ Btu per (hr) (sq ft) (F deg).}$$

The transfer area on which this conductance is based is the inside tube area. Associated with 1 ft length of pipe there are:

$$\pi \times \frac{2.067}{12} \times 1 = 0.542 \text{ sq ft.}$$

Thus the resistance for 1 ft of tube length is:

$$R_1 = \frac{1}{h_c \pi D \times 1} = \frac{1}{650 \times 0.542} = 2.8 \times 10^{-3} \text{ hr Fahrenheit degree per Btu.}$$

Case 9, Table 5 is applicable for calculating the free thermal convection resistance, R_c, existing between the surrounding air and the insulation. The air temperature is given as 120 F. As an approximation a 20 deg temperature difference between the air and the pipe surface will be assumed. $D = 4.375/12 = 0.364$ ft. Then case 9 yields:

$$h_c = 0.23 \left(\frac{20}{0.364}\right)^{0.25} = 0.63 \text{ Btu per (hour) (square foot) (Fahrenheit degree).} \quad (13)$$

This result may not be deemed conservative inasmuch as the expression is for *still* air. If, however, the air is not still, but flows at approximately 5 mph or 7 fps the mass velocity corresponds to:

$$G = 7 \times 0.07 \times 3600 = 1770 \text{ lb air per (hour) (square foot).}$$

A magnitude of $k = 0.014$ Btu per (hour) (square foot) (Fahrenheit degree) per one foot thickness applied to case 4 yields:

$$h_c = 0.45\left(\frac{0.014}{0.364}\right) + 0.178\,(1770)^{0.56}\left(\frac{0.014}{0.364}\right)^{0.44}$$

$$= 0.017 + 2.8 = 2.8 \text{ Btu per (hour) (square foot) (Fahrenheit degree).}$$

This conductance is based on 1 sq ft of outside lagging area. Thus, since there are $\pi \times (4.375/12) = 1.14$ sq ft of outside lagging area associated with 1 ft length of pipe:

$$R_c = \frac{1}{2.8 \times 1.14} = 0.312 \text{ hr Fahrenheit degree per Btu.}$$

The radiation resistance, R_r, which acts in parallel with the convection resistance, R_c, for the transfer of heat to the surface of the insulation, may be calculated. For the purposes of this illustrative problem it will be assumed that the insulated pipe is exposed to (*sees*) surroundings, which exist at 120 F. Then the angle factor, F_A, is unity and for an estimated surface emissivity of 0.9 (see Table 6), $F_e = 0.9$. As a first approximation the insulation surface temperature will be estimated as 20 deg below the surroundings at 120 F. Then the radiation per degree of temperature difference, by Equation 3 (or more conveniently by Table 8) divided by the temperature difference will be:

$$h_r = \frac{(196 - 170)\,0.95}{20} = 1.17 \text{ Btu per (hour) (square foot) (Fahrenheit degree).}$$

The outside surface area of the insulation associated with 1 ft of pipe length was previously calculated as 1.14 sq ft. Thus:

$$R_r = \frac{1}{1.17 \times 1.14} = 0.75 \text{ hr Fahrenheit degree per Btu.}$$

The resultant resistance of R_c and R_r acting in parallel (see Fig. 4) can now be evaluated as:

$$\frac{1}{R_4} = \frac{1}{R_c} + \frac{1}{R_r} = \frac{1}{0.312} + \frac{1}{0.75} = 4.54 \text{ Btu per (hour) (Fahrenheit degree).}$$

$$R_4 = 0.22 \text{ hr Fahrenheit degree per Btu.}$$

The over-all resistance, R_T, surroundings to cold water, is the sum of $R_1 + R_2 + R_3 + R_4 = 4.1$ hr F deg per Btu for 1 ft length of pipe. Note that the controlling resistances are R_3 and R_4 and that neglect of both R_1 and R_2 would not significantly influence the total resistance, R_T.

On the basis of this resistance calculation the heat transfer from the surroundings to the cold water may be evaluated as:

$$\frac{q_{rc}}{N} = \frac{t_o - t_f}{R_T} = \frac{120 - 34}{4.1} = 21 \text{ Btu per (hour) (foot)}$$

or about 0.175 tons of refrigeration per 100 ft of pipe.