The curves in Fig. 4 may be approximated very closely by the empirical formula:[1]

$$f = .0055\left[1 + \left(20{,}000\,\frac{e}{d} + \frac{10^6}{N_{Re}}\right)^{1/3}\right] \tag{12}$$

Equation 8 is applicable to all liquids, and to gases when the pressure loss is less than 10 percent of the initial pressure. When the loss in head is high, the formula to be used for gases is

$$\frac{p_1^2 - p_2^2}{p_1^2} = \frac{flV_1^2}{gd\,p_1v_1} \tag{13}$$

which may be arranged to give the loss in pressure,

$$p_1 - p_2 = p_1\left[1 - \sqrt{1 - \frac{flV_1^2}{gd\,p_1v_1}}\right] \tag{14}$$

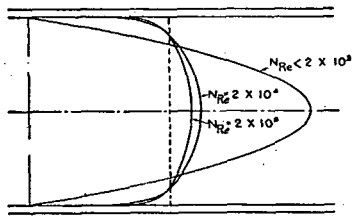

FIG. 5. COMPARISON OF VELOCITY PROFILES FOR 3 DIFFERENT REYNOLDS NUMBERS BUT FOR SAME AVERAGE VELOCITY

Pressure Loss in Non-Circular Pipes

The formulas for friction loss in pipes are based on the use of pipes of circular cross-section. The same formulas may be extended to non-circular sections, by suitable modification. In the basic formula, Equation 8, the internal diameter d is to be replaced by the hydraulic diameter d_H defined by the equation:

$$d_H = \frac{4 \times \text{area of cross-section}}{\text{wetted perimeter of cross-section}} \tag{15}$$

For example, in a rectangular duct, 1 ft by 2 ft, the cross-section area is 2 sq ft, and the perimeter 6 ft. Then the hydraulic diameter will be $d_H = (4 \times 2)/6 = 1^1/_3$ ft.

In the case of a round pipe,

$$d_H = \frac{4 \times \pi d^2/4}{\pi d} = d \tag{16}$$

In computing the Reynolds number, and from that the friction factor, the hydraulic diameter is not to be used. A better approximate procedure is to replace the length in the Reynolds number by the shortest dimension plus one-fourth of the hydraulic diameter. Thus, in a duct of dimension a x b where a < b, N_{Re}, for the purposes of calculating friction factors, is

$$N_{Re} = (a + 0.25d_H)V\rho/\mu \tag{17}$$

TABLE 1. VALUES OF e FOR DIFFERENT KINDS OF PIPE

TYPE OF PIPE	e
Smooth drawn tubing	0.000005
Commercial steel or wrought iron	0.00015
Asphalted cast-iron	0.0004
Galvanized iron	0.0005
Cast-iron	0.00085
Wood stave	0.0006 to 0.003
Concrete	0.001 to 0.01
Riveted steel	0.003 to 0.03

This value of N_{Re} may be used in Equation 10 for laminar flow, and in Equation 12 or Fig. 4 for turbulent flow. The error in the approximation is somewhat greater for laminar than for turbulent flow. In the former case, the relative error may be as much as 10 percent, while in the latter it almost always is less than 3 percent.

FLOW OF COMPRESSIBLE FLUIDS

In the flow of compressible fluids, the large density variations make impracticable the use of the Bernoulli equation, (Equation 7). In certain special cases, however, the exact equations for compressible flow may be stated. If flow occurs with no friction or other internal irreversibility, Equation 6 becomes

$$\frac{1}{2g}\,dV^2 + \frac{dp}{\rho} = 0 \tag{18}$$

If, in addition, the flow is adiabatic,

$$p\rho^{-k} = p_1\rho_1^{-k} \tag{19}$$

so that Equation 18 becomes

$$\frac{1}{2g}\,dV^2 + \frac{p_1^{1/k}}{\rho_1}\frac{dp}{\rho^{1/k}} = 0 \tag{20}$$

or by integration,

$$\frac{1}{2g}(V_2^2 - V_1^2) + \frac{k}{k-1}\frac{p_1}{\rho_1}\left[\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} - 1\right] = 0 \tag{21}$$

This extension to compressible flow of Bernoulli's equation reduces to the more familiar form if the pressure change is small.

The ratio of specific heats, k, is used extensively in fluid dynamics; values of k for various gases are given in Table 2.

TABLE 2. RATIO OF SPECIFIC HEAT AT CONSTANT PRESSURE TO SPECIFIC HEAT AT CONSTANT VOLUME FOR COMPRESSIBLE FLUIDS

COMPRESSIBLE FLUID	RATIO $k = c_p/c_v$
Helium and other monatomic gases	1.66
Air and other diatomic gases	1.40
Ammonia and hydrogen sulfide	1.34
Carbon dioxide, methane, natural gas, superheated steam, moist steam down to a quality of 97 percent	1.28 to 1.32
Sulfur dioxide, ethylene, acetylene	1.24 to 1.26